MATH 133 – Calculus with Fundamentals 1
Problem Set 7B Solutions, November 17, 2017

Section 3.8

70. To find $\frac{dy}{dx}$, we use implicit differentiation:

$$2(x^2 + y^2 - 4x)(2x + 2y\frac{dy}{dx} - 4) = 4x + 4y\frac{dy}{dx}$$

So solving for $\frac{dy}{dx}$,

$$\frac{dy}{dx} = \frac{4x + 2(4 - 2x)(x^2 + y^2 - 4x)}{4y(x^2 + y^2 - 4x) - 4y}.$$

The four $y$-coordinates for the intersections with the line $x = 1$ are found by substituting into the original equation:

$$(y^2 - 3)^2 = 2(1 + y^2)$$

and then rearranging and solving for $y$:

$$y^4 - 6y^2 + 9 = 2 + 2y^2, \quad \text{so} \quad y^4 - 8y^2 + 7 = 0.$$

This factors as

$$(y^2 - 1)(y^2 - 7) = 0$$

so the solutions are $y = \pm 1$, $y = \pm\sqrt{7}$. Then at $(x, y) = (1, 1)$ for instance, we have

$$\frac{dy}{dx} = \frac{4 + 2(2)(-2)}{4(-2) - 4} = \frac{1}{3}$$

so the tangent line at $(1, 1)$ is

$$y - 1 = \frac{1}{3}(x - 1).$$

The other three are found similarly:

- At $(x, y) = (1, -1)$, $\frac{dy}{dx} = \frac{-1}{3}$ and the tangent line is

$$y + 1 = \frac{-1}{3}(x - 1)$$

- At $(x, y) = (1, \sqrt{7})$, $\frac{dy}{dx} = \frac{5\sqrt{7}}{21}$ (after simplifying), so the tangent line is

$$y - \sqrt{7} = \frac{5\sqrt{7}}{21}(x - 1)$$

- Finally, at $(x, y) = (1, -\sqrt{7})$, $\frac{dy}{dx} = -\frac{5\sqrt{7}}{21}$, and the tangent line is

$$y + \sqrt{7} = -\frac{5\sqrt{7}}{21}(x - 1).$$

Section 3.10

16. In the figure given with the problem, let $x$ be the distance from the intersection to the car as a function of $t$ and let $z$ be the distance from the car to the house. By the Pythagorean theorem,

$$2^2 + x^2 = z^2.$$

So taking the derivative with respect to $t$ (implicitly)

$$2x\frac{dx}{dt} = 2z\frac{dz}{dt}.$$

At the instant of time the problem is asking about, $x = 6$ and $\frac{dx}{dt} = 80$. From the Pythagorean Theorem, $4 + 36 = 40 = z^2$, so $z = \sqrt{40} \doteq 6.32$. Hence

$$12 \cdot 80 \doteq 12.64\frac{dz}{dt} \Rightarrow \frac{dz}{dt} \doteq \frac{960}{12.64} \doteq 75.95$$

(units are km/hr).