MATH 133 – Calculus with Fundamentals 1
Problem Set 5B Solutions, October 27, 2017

Section 3.1

51. The derivative is positive on the intervals $1.0 < x < 2.5$ and again on $x > 3.5$. These are the points on the graph where the slope of the tangent line is *positive*. The slopes of the tangent lines are negative or zero at all other points.

52. The graph is in Figure 1 at the top of the next page (for typographical reasons). $x = \pi/2$ is at the midpoint of the interval, where the tangent line to $y = f(x) = \sin(x)$ appears to be a *horizontal* line. Hence we expect $f'(\pi/2) = 0$. We check this by taking the difference quotient

$$\frac{f(\pi/2 + h) - f(\pi/2)}{h}$$

for $h = 0.001$ and $h = -0.001$:

$$\frac{\sin(\pi/2 + 0.001) - \sin(\pi/2)}{.001} \doteq -0.0005$$

and

$$\frac{\sin(\pi/2 - 0.001) - \sin(\pi/2)}{-0.001} \doteq 0.0005$$

Note that both are close to zero. Moreover, the difference quotient for $h = +0.001$ gives a negative value, while $h = -0.001$ gives a positive value. This is expected from the graph, since the values above are slopes of secant lines. The one with $h > 0$ is sloping slightly downward because $f(x)$ is starting to decrease from the maximum at $x = \pi/2$, while the one with $h < 0$ is positive, because that secant line is sloping upward.

Section 3.2

44. Looking at Figure 14 in the text, we see $y = g(x) < 0$ when $y = f(x)$ is sloping down, while $y = g(x) > 0$ when $y = f(x)$ is sloping up. Moreover $g(x) = 0$ at the same $x = 0$ where $f(x)$ has a minimum. Therefore $g(x) = f'(x)$.

46.

(a) See plot in Figure 2 which gives the rough shape of $Q'(t)$. $Q'(t)$ represents the rate of change of the cumulative total amount of crude oil as a function of time.

(b) The maximum derivative occurs roughly at $t = 2005$ (about 1/5 of the way between $t = 2000$ and $t = 2025$). The question did not ask for this, but I estimated the value $Q'(2005)$ to generate the plot in Figure 2 like this: Draw in a line tangent to the graph at $t = 2005$ by hand and see where that line crosses the lines $Q = 0.5$ and $Q = 1.5$. That happens roughly at $t = 1985$ and $t = 2020$, so the slope of the tangent is approximately

$$Q'(2005) \doteq \frac{1.5 - 0.5}{2020 - 1985} \doteq .029$$

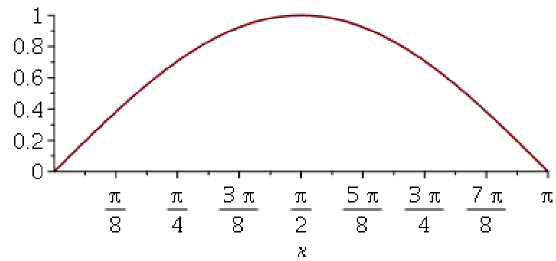

Figure 1: $y = \sin(x)$ on $[0, \pi]$.

(units are trillions of barrels per year).

(c) $\lim_{t\to\infty} Q(t) = 2.3$ trillion barrels. That is the total cumulative amount of crude oil produced over all time.

(d) $\lim_{t\to\infty} Q'(t) = 0$, because the slope of the graph $Q(t)$ goes to zero as t increases without bound. The interpretation is that eventually, the cumulative amount of crude oil produced will not be changing any more. *We will have found and used up all of the easily accessible crude oil, so no more will be produced.*

66.

(A) has derivative (III) (Note: (A) has three points where the derivative is zero; (III) is the only graph that crosses the x-axis three times)

(B) has derivative (I)

(C) has derivative (II)

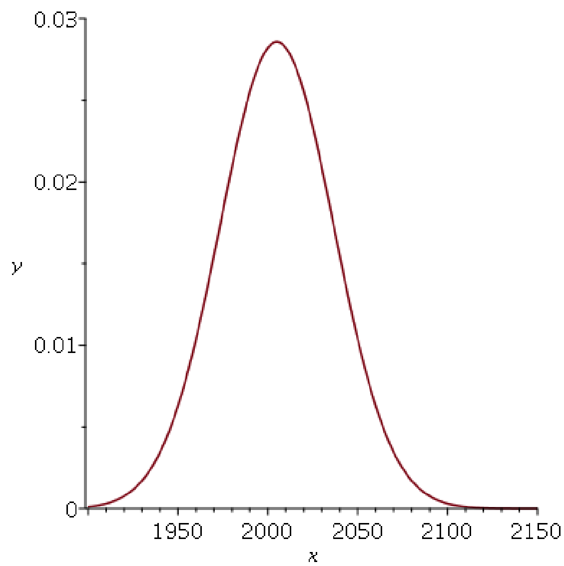

Figure 2: $Q'(t)$ for Problem 46.