MATH 133 – Calculus with Fundamentals 1
Problem Set 4B Solutions, October 6, 2017

Section 2.3

40. By the Limit Sum Law,

$$\lim_{x\to-4}(2f(x)+3g(x)) = \lim_{x\to-4} 2f(x) + \lim_{x\to-4} 3g(x)$$

Then by the Limit Product Law and the given information, this equals

$$\begin{aligned}&= (\lim_{x\to-4} 2)\cdot(\lim_{x\to-4} f(x)) + (\lim_{x\to-4} 3)\cdot(\lim_{x\to-4} g(x))\\ &= 2\cdot 3 + 3\cdot 1\\ &= 9.\end{aligned}$$

Section 2.4

64. Any graph $y = f(x)$ for which

- $\lim_{x\to2^-} f(x) = f(2)$, but $\lim_{x\to2^+} f(x)$ is something other than $f(2)$ (it could be a different number – so a jump discontinuity at $x = 2$ or even an infinite discontinuity from the right side), and
- $\lim_{x\to3^+} f(x) = f(3)$, but $\lim_{x\to3^-} f(x)$ is something other than $f(3)$ (it could be a different number – so a jump discontinuity at $x = 2$ or even an infinite discontinuity from the right side).

is OK. One example is shown in Figure 1.

66. This one is similar to 64 – any graph for which

- $\lim_{x\to1^+} f(x) = f(1)$, but $\lim_{x\to1^-} f(x)$ is something other than $f(1)$, and
- $\lim_{x\to2^-} f(x) = f(2)$, but $\lim_{x\to2^+} f(x)$ is something other than $f(2)$,
- $\lim_{x\to3^-} f(x)$ and $\lim_{x\to3^+} f(x)$ both different from $f(3)$. This could be because at least one of the one-sided limit does not exist (e.g. infinite discontinuities), or because $f(x)$ has a removable singularity at $x = 3$, with $f(3)$ different from the one-sided limits.)

is OK. One example is shown in Figure 2.

Section 2.5/

8. Factor the top and cancel $x - 8$ to get a determinate limit:

$$\begin{aligned}\lim_{x\to8}\frac{x^3-64x}{x-8} &= \lim_{x\to8}\frac{x(x+8)(x-8)}{(x-8)}\\ &= \lim_{x\to8} x(x+8) = 128.\end{aligned}$$

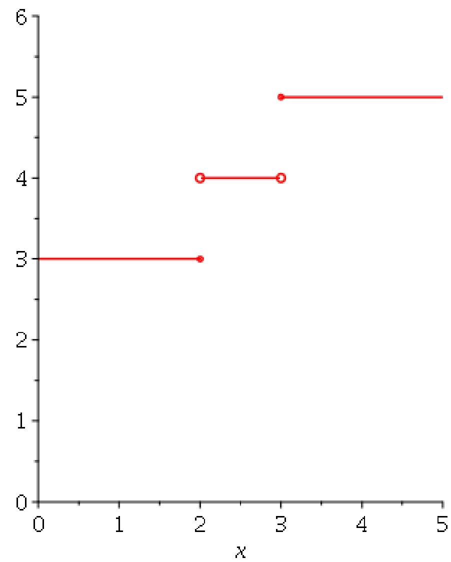

Figure 1: $y = f(x)$ for Question 2.4/64.

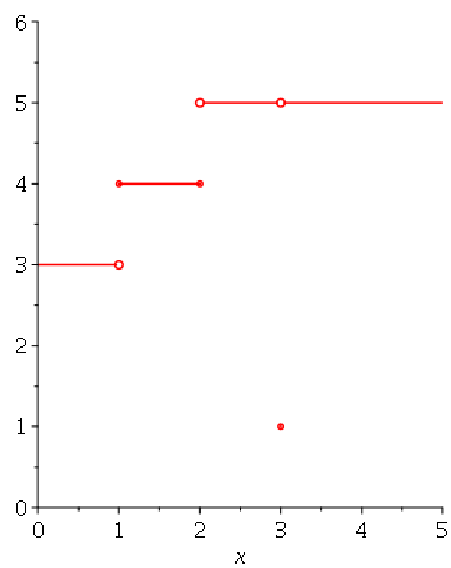

Figure 2: $y = f(x)$ for Question 2.4/66.

19. Put the terms on the top over a common denominator, simplify the fraction, and work to cancel a factor of h top and bottom:

$$\begin{aligned}
\lim_{h\to 0} \frac{\frac{1}{(h+2)^2} - \frac{1}{4}}{h} = \lim_{h\to 0} \frac{\frac{4-(h+2)^2}{4(h+2)^2}}{h} \\
&= \lim_{h\to 0} \frac{4-(h+2)^2}{4h(h+2)^2} \\
&= \lim_{h\to 0} \frac{4-(h^2+4h+4)}{4h(h+2)^2} \\
&= \lim_{h\to 0} \frac{-h(h+4)}{4h(h+2)^2} \qquad \text{(now cancel } h\text{)} \\
&= \lim_{h\to 0} \frac{-(h+4)}{4(h+2)^2} \\
&= \frac{-1}{4}.
\end{aligned}$$

22. Multiply by the conjugate radical top and bottom, simplify, cancel $x-8$ and evaluate:

$$\begin{aligned}
\lim_{x\to 8} \frac{\sqrt{x-4}-2}{x-8} &= \lim_{x\to 8} \frac{(\sqrt{x-4}-2)(\sqrt{x-4}+2)}{(x-8)(\sqrt{x-4}+2)} \\
&= \lim_{x\to 8} \frac{(x-4)-4}{(x-8)(\sqrt{x-4}+2)} \\
&= \lim_{x\to 8} \frac{(x-8)}{(x-8)(\sqrt{x-4}+2)} \\
&= \lim_{x\to 8} \frac{1}{\sqrt{x-4}+2} \\
&= \frac{1}{4}.
\end{aligned}$$