MATH 133 – Calculus with Fundamentals 1
Problem Set 2B Solutions, September 15, 2017

Note: Because of the confusion in class on Wednesday, credit was given for either 1.3/39 *or* 1.4/39. Solutions for *both* are shown below.

1.3/30. With $f(x) = |x|$ and $g(\theta) = \sin(\theta)$,

$$(f \circ g)(\theta) = f(g(\theta)) = |\sin(\theta)|.$$

The domain is the set of all real θ because any such θ is in the domain of g and then $g(\theta)$ is in the domain of f. On the other hand,

$$(g \circ f)(x) = \sin(|x|)$$

The domain is the set of all real x because any such x is in the domain of f and then $f(x)$ is in the domain of g.

1.3/36. When $x < 0$, we get a portion of a line of slope 1 and y-intercept $+1$; when $x \geq 0$, we get a portion of a line of slope -1 and y-intercept $+1$. The graph is shown in Figure 1.

1.3/39. Starting at $t = t_0$, the time 10 years later is $t = t_0 + 10$. We have, using rules for exponents:

$$\begin{aligned} P(t_0 + 10) &= 30 \cdot 2^{(0.1)(t_0+10)} \\ &= 30 \cdot 2^{(0.1)t_0+1} \\ &= 30 \cdot 2^{(0.1)t_0} \cdot 2 \\ &= 2P(t_0). \end{aligned}$$

This shows the population doubles in the time from t_0 to $t_0 + 10$.

$$\begin{aligned} P(t_0 + 1/k) &= a \cdot 2^{k(t_0+1/k)} \\ &= a \cdot 2^{kt_0+1} \\ &= a \cdot 2^{kt_0} \cdot 2 \\ &= 2P(t_0). \end{aligned}$$

So the population doubles after $1/k$ years.

1.4/39. This looks like the cos graph, but with amplitude 3 and period 4π, so an equation would be

$$y = 3\cos(x/2)$$

(there is no vertical shift in this case). There are other correct ways to write this of course. For instance if you think of the part of the graph between $x = -\pi$ and $x = 3\pi$ as the period, then its a horizontally shifted and vertically stretched sine graph with equation

$$y = 3\sin((x + \pi)/2).$$

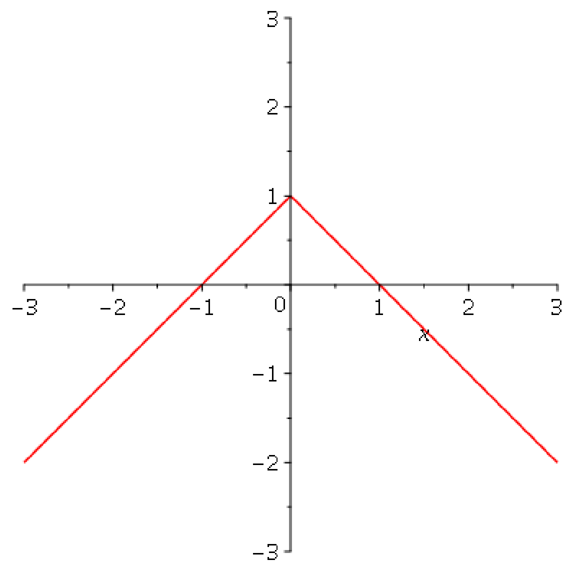

Figure 1: The graph $y = f(x)$ in 1.3/36

1.4/40. I saw this first as another cosine graph, but reflected across the x-axis. The amplitude is 2 and the period is $4\pi/3$, so

$$y = -2\cos(3x/2)$$

is one possible equation. There are others too as in the solution for 39 above.

1.4/59. By the Law of Cosines,

$$PQ^2 = 10^2 + 8^2 - 2 \cdot 10 \cdot 8 \cdot \cos(7\pi/9) \doteq 100 + 64 - 160 \cdot (-.76604) \doteq 286.57$$

So $PQ \doteq \sqrt{286.57} \doteq 16.928$.