61. Determine whether the function is even, odd, or neither.

(a) $f(t) = \dfrac{1}{t^4+t+1} - \dfrac{1}{t^4-t+1}$ **(b)** $g(t) = 2^t - 2^{-t}$

(c) $G(\theta) = \sin\theta + \cos\theta$ **(d)** $H(\theta) = \sin(\theta^2)$

62. Write $f(x) = 2x^4 - 5x^3 + 12x^2 - 3x + 4$ as the sum of an even and an odd function.

63. Determine the values of x for which $f(x) = \dfrac{1}{x-4}$ is increasing and for which decreasing.

64. State whether the function is increasing, decreasing, or neither.

(a) Surface area of a sphere as a function of its radius

(b) Temperature at a point on the equator as a function of time

(c) Price of an airline ticket as a function of the price of oil

(d) Pressure of the gas in a piston as a function of volume

In Exercises 65–70, let f be the function shown in Figure 27.

65. Find the domain and range of f.

66. Sketch the graphs of $y = f(x+2)$ and $y = f(x) + 2$.

67. Sketch the graphs of $y = f(2x)$, $y = f\left(\frac{1}{2}x\right)$, and $y = 2f(x)$.

68. Sketch the graphs of $y = f(-x)$ and $y = -f(-x)$.

69. Extend the graph of f to $[-4, 4]$ so that it is an even function.

70. Extend the graph of f to $[-4, 4]$ so that it is an odd function.

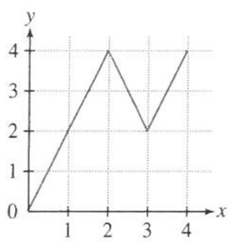

FIGURE 27

71. Suppose that f has domain $[4, 8]$ and range $[2, 6]$. Find the domain and range of:

(a) $y = f(x) + 3$ **(b)** $y = f(x+3)$

(c) $y = f(3x)$ **(d)** $y = 3f(x)$

72. Let $f(x) = x^2$. Sketch the graph over $[-2, 2]$ of:

(a) $y = f(x+1)$ **(b)** $y = f(x) + 1$

(c) $y = f(5x)$ **(d)** $y = 5f(x)$

73. Suppose that the graph of $f(x) = \sin x$ is compressed horizontally by a factor of 2 and then shifted 5 units to the right.

(a) What is the equation for the new graph?

(b) What is the equation if you first shift by 5 and then compress by 2?

(c) GU Verify your answers by plotting your equations.

74. Figure 28 shows the graph of $f(x) = |x| + 1$. Match the functions (a)–(e) with their graphs (i)–(v).

(a) $y = f(x-1)$ **(b)** $y = -f(x)$ **(c)** $y = -f(x) + 2$

(d) $y = f(x-1) - 2$ **(e)** $y = f(x+1)$

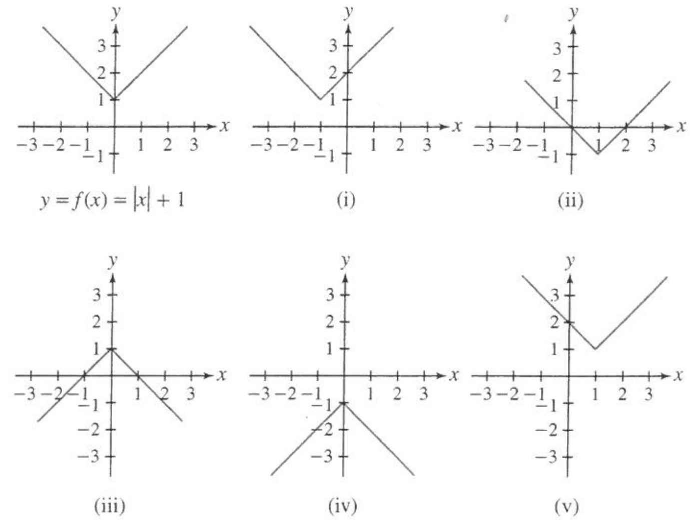

FIGURE 28

75. Sketch the graph of $y = f(2x)$ and $y = f\left(\frac{1}{2}x\right)$, where $f(x) = |x| + 1$ (Figure 28).

76. Find the function f whose graph is obtained by shifting the parabola $y = x^2$ by 3 units to the right and 4 units down, as in Figure 29.

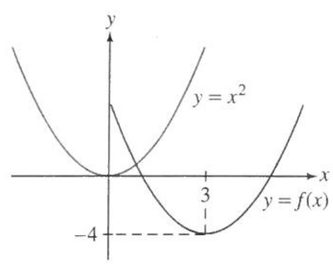

FIGURE 29

77. Define $f(x)$ to be the larger of x and $2 - x$. Sketch the graph of f. What are its domain and range? Express $f(x)$ in terms of the absolute value function.

78. For each curve in Figure 30, state whether it is symmetric with respect to the y-axis, the origin, both, or neither.

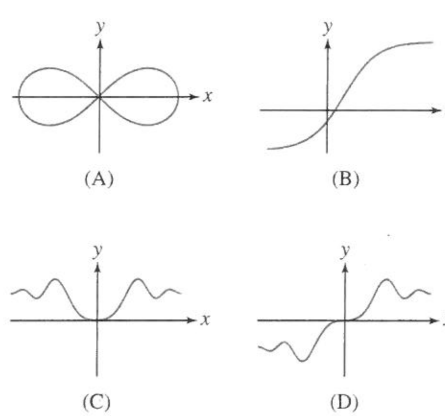

FIGURE 30

79. Show that the sum of two even functions is even and the sum of two odd functions is odd.

Exercises

In Exercises 1–4, find the slope, the y-intercept, and the x-intercept of the line with the given equation.

1. $y = 3x + 12$

2. $y = 4 - x$

3. $4x + 9y = 3$

4. $y - 3 = \frac{1}{2}(x - 6)$

In Exercises 5–8, find the slope of the line.

5. $y = 3x + 2$

6. $y = 3(x - 9) + 2$

7. $3x + 4y = 12$

8. $3x + 4y = -8$

In Exercises 9–20, find the equation of the line with the given description.

9. Slope 3, y-intercept 8

10. Slope -2, y-intercept 3

11. Slope 3, passes through $(7, 9)$

12. Slope -5, passes through $(0, 0)$

13. Horizontal, passes through $(0, -2)$

14. Passes through $(-1, 4)$ and $(2, 7)$

15. Parallel to $y = 3x - 4$, passes through $(1, 1)$

16. Passes through $(1, 4)$ and $(12, -3)$

17. Perpendicular to $3x + 5y = 9$, passes through $(2, 3)$

18. Vertical, passes through $(-4, 9)$

19. Horizontal, passes through $(8, 4)$

20. Slope 3, x-intercept 6

21. Find the equation of the perpendicular bisector of the segment joining $(1, 2)$ and $(5, 4)$ (Figure 12). *Hint:* The midpoint Q of the segment joining (a, b) and (c, d) is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$.

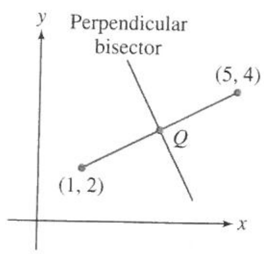

FIGURE 12

22. Intercept-Intercept Form Show that if $a, b \neq 0$, then the line with x-intercept $x = a$ and y-intercept $y = b$ has equation (Figure 13)

$$\frac{x}{a} + \frac{y}{b} = 1$$

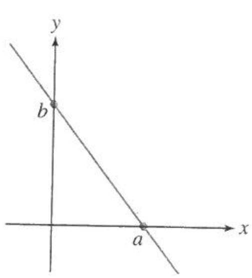

FIGURE 13

23. Find an equation of the line with x-intercept $x = 4$ and y-inte $y = 3$.

24. Find y such that $(3, y)$ lies on the line of slope $m = 2$ thı $(1, 4)$.

25. Determine whether there exists a constant c such that the $x + cy = 1$:

(a) has slope 4.

(b) passes through $(3, 1)$.

(c) is horizontal.

(d) is vertical.

26. Assume that the number N of concert tickets that can be sol price of P dollars per ticket is a linear function $N(P)$ for $10 \leq P$ Determine $N(P)$ (called the demand function) if $N(10) = 500$ $N(40) = 0$. What is the decrease ΔN in the number of tickets sc the price is increased by $\Delta P = 5$ dollars?

27. Suppose that the number of a certain type of computer that be sold when its price is P (in dollars) is given by a linear func $N(P)$. Determine $N(P)$ if $N(1000) = 10{,}000$ and $N(1500) = 7,$ What is the change ΔN in the number of computers sold if the pri increased by $\Delta P = 100$ dollars?

28. Suppose that the demand for Colin's kidney pies is linear in price P. Determine the demand function N as a function of P gi the number of pies sold when the price is P if he can sell 100 pies w the price is \$5.00 and he can sell 40 pies when the price is \$10.00. termine the revenue $(N \times P)$ for prices $P = 5, 6, 7, 8, 9, 10$ and t choose a price to maximize the revenue.

29. Materials expand when heated. Consider a metal rod of length at temperature T_0. If the temperature is changed by an amount ΔT, t the rod's length approximately changes by $\Delta L = \alpha L_0 \Delta T$, where the thermal expansion coefficient and ΔT is not an extreme temperat change. For steel, $\alpha = 1.24 \times 10^{-5}\,{}^\circ\mathrm{C}^{-1}$.

(a) A steel rod has length $L_0 = 40$ cm at $T_0 = 40°$C. Find its len at $T = 90°$C.

(b) Find its length at $T = 50°$C if its length at $T_0 = 100°$C is 65 c

(c) Express length L as a function of T if $L_0 = 65$ cm at $T_0 = 100$

30. Do the points $(0.5, 1)$, $(1, 1.2)$, $(2, 2)$ lie on a line?

31. Find b such that $(2, -1)$, $(3, 2)$, and $(b, 5)$ lie on a line.

32. Find an expression for the velocity v as a linear function of t tl matches the following data:

t (s)	0	2	4	6
v (m/s)	39.2	58.6	78	97.4

33. The period T of a pendulum is measured for pendulums of seve different lengths L. Based on the following data, does T appear to a linear function of L?

L (cm)	20	30	40	50
T (s)	0.9	1.1	1.27	1.42

34. Show that f is linear of slope m if and only if

$$f(x + h) - f(x) = mh \quad \text{(for all } x \text{ and } h\text{)}$$

That is to say, prove the following two statements:

(a) f is linear of slope m implies that $f(x + h) - f(x) = m$ (for all x and h).

(b) $f(x+h) - f(x) = mh$ (for all x and h) implies that f is linear of slope m.

35. Find the roots of the quadratic polynomials:
(a) $f(x) = 4x^2 - 3x - 1$ **(b)** $f(x) = x^2 - 2x - 1$

In Exercises 36–43, complete the square and find the minimum or maximum value of the quadratic function.

36. $y = x^2 + 2x + 5$

37. $y = x^2 - 6x + 9$

38. $y = -9x^2 + x$

39. $y = x^2 + 6x + 2$

40. $y = 2x^2 - 4x - 7$

41. $y = -4x^2 + 3x + 8$

42. $y = 3x^2 + 12x - 5$

43. $y = 4x - 12x^2$

44. Sketch the graph of $y = x^2 - 6x + 8$ by plotting the roots and the minimum point.

45. Sketch the graph of $y = x^2 + 4x + 6$ by plotting the minimum point, the y-intercept, and one other point.

46. If the alleles A and B of the cystic fibrosis gene occur in a population with frequencies p and $1 - p$ (where p is a fraction between 0 and 1), then the frequency of heterozygous carriers (carriers with both alleles) is $2p(1-p)$. Which value of p gives the largest frequency of heterozygous carriers?

47. For which values of c does $f(x) = x^2 + cx + 1$ have a double root? No real roots?

48. Let f be a quadratic function and c a constant. Which of the following statements is correct? Explain graphically.
(a) There is a unique value of c such that $y = f(x) - c$ has a double root.
(b) There is a unique value of c such that $y = f(x - c)$ has a double root.

49. Prove that $x + \frac{1}{x} \geq 2$ for all $x > 0$. *Hint:* Consider $(x^{1/2} - x^{-1/2})^2$.

50. Let $a, b > 0$. Show that the *geometric mean* $\sqrt{ab}$ is not larger than the *arithmetic mean* $(a+b)/2$. *Hint:* Use a variation of the hint given in Exercise 49.

51. If objects of weights x and w_1 are suspended from the balance in Figure 14(A), the cross-beam is horizontal if $bx = aw_1$. If the lengths a and b are known, we may use this equation to determine an unknown weight x by selecting w_1 such that the cross-beam is horizontal. If a and b are not known precisely, we might proceed as follows. First balance x by w_1 on the left as in (A). Then switch places and balance x by w_2 on the right as in (B). The average $\bar{x} = \frac{1}{2}(w_1 + w_2)$ gives an estimate for x. Show that $\bar{x}$ is greater than or equal to the true weight x.

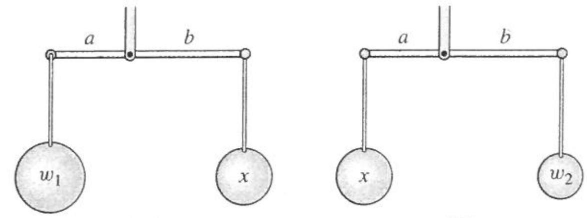

FIGURE 14

52. Find numbers x and y with sum 10 and product 24. *Hint:* Find a quadratic polynomial satisfied by x.

53. Find a pair of numbers whose sum and product are both equal to 8.

54. Show that the parabola $y = x^2$ consists of all points P such that $d_1 = d_2$, where d_1 is the distance from P to $\left(0, \frac{1}{4}\right)$ and d_2 is the distance from P to the line $y = -\frac{1}{4}$ (Figure 15).

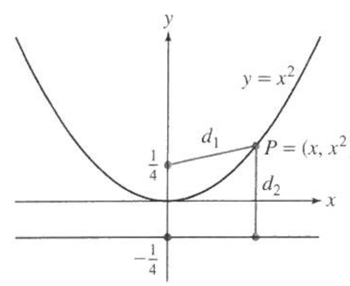

FIGURE 15

Further Insights and Challenges

55. Show that if f and g are linear, then so is $f + g$. Is the same true of fg?

56. Show that if f and g are linear functions such that $f(0) = g(0)$ and $f(1) = g(1)$, then $f = g$.

57. Show that $\Delta y / \Delta x$ for the function $f(x) = x^2$ over the interval $[x_1, x_2]$ is not a constant, but depends on the interval. Determine the exact dependence of $\Delta y / \Delta x$ on x_1 and x_2.

58. Complete the square and use the result to derive the quadratic formula for the roots of $ax^2 + bx + c = 0$.

59. Let $a, c \neq 0$. Show that the roots of

$$ax^2 + bx + c = 0 \quad \text{and} \quad cx^2 + bx + a = 0$$

are reciprocals of each other.

60. Show, by completing the square, that the parabola

$$y = ax^2 + bx + c$$

is congruent to $y = ax^2$ by a vertical and horizontal translation.

61. Prove **Viète's Formulas**: The quadratic polynomial with α and β as roots is $x^2 + bx + c$, where $b = -\alpha - \beta$ and $c = \alpha\beta$.

1.3 The Basic Classes of Functions

It would be impossible (and useless) to describe all possible functions f. Since the values of a function can be assigned arbitrarily, a function chosen at random would likely be so complicated that we could neither graph it nor describe it in any reasonable way. However, calculus makes no attempt to deal with all functions. The techniques of calculus, powerful