MATH 133 – Calculus with Fundamentals 1
Exam 4 Sample/Practice Problems
November 28, 2017

Disclaimer: As always, the actual exam questions may be posed in different ways, may be formatted differently, and may focus on slightly different aspects of the material we have covered. Also, the actual exam will be significantly shorter than this group of questions(!)

I. Find y'; don't simplify.

(A)

$$y = \ln(x)\left(x^7 - \frac{4}{\sqrt{x}}\right)$$

(B)

$$y = \sin^{-1}(e^{2x} + 2)$$

(C)

$$y = \frac{\ln(x+1)}{3x^4 - 1}$$

(D)

$$y = \frac{\sin(x)}{1 + \cos(x)}$$

(E)

$$y = \tan^{-1}(x^2 + x)$$

(F) Using implicit differentiation:

$$xy^2 - 3y^3 + 2x^4 - 4xy = 2$$

(G) Find the equation of the line tangent to the curve from (F) at $(x, y) = (1, 0)$

II. The quantity of a reagent present in a chemical reaction is given by $Q(t) = t^3 - 3t^2 + t + 30$ grams at time $t \geq 0$ seconds.

(A) Over which intervals with $t \geq 0$ is the amount increasing? decreasing?

(B) Over which intervals is the rate of change of Q increasing? decreasing?

III. A spherical balloon is being inflated at 20 cubic inches per minute. (The volume of a sphere of radius r is $V = \frac{4\pi r^3}{3}$ and the surface area is $V = 4\pi r^2$.)

(A) When the radius is 6 inches, at what rate is the radius of the balloon increasing?

(B) When the radius is 6 inches, at what rate is the surface area increasing?

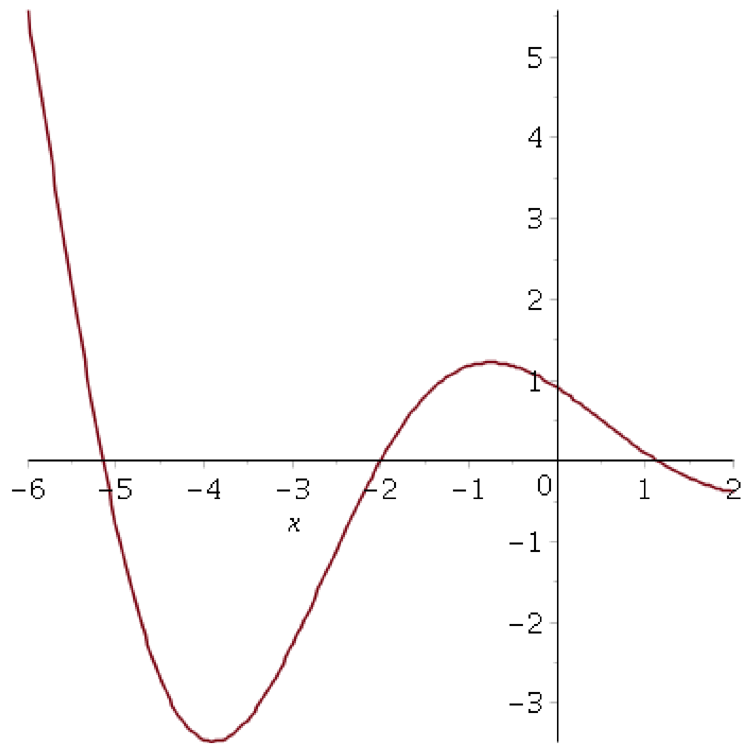

Figure 1: Plot of $y = f'(x)$ for Problem VI

IV. A baseball diamond is a square with side of length 90 feet. After hitting the ball, a player leaves home plate and runs toward first base at 15 ft/sec. (Assume the runner is running straight along the base path – this is a bit unrealistic, of course, but let's keep it simple for the purposes of the problem!) How fast is the runner's distance from second base changing when he is half way to first base?

V. All parts of this question refer to $f(x) = 4x^3 - x^4$.

(A) Find and classify all the critical points of f using the First Derivative Test.

(B) Over which intervals is the graph $y = f(x)$ concave up? concave down?

(C) Sketch the graph $y = f(x)$.

(D) Find the absolute maximum and minimum of $f(x)$ on the interval $[1, 4]$.

VI. All three parts of this question refer to the function $f(x)$ *whose derivative is plotted in Figure 1. NOTE: This is the graph* $y = f'(x)$ *not* $y = f(x)$.

(A) Give approximate values for all the critical points of $f(x)$ in the interval shown, and say whether f has a local maximum, a local minimum, or neither at each.

(B) Find approximate values for all the inflection points of $f(x)$.

(C) Over which intervals is $y = f(x)$ concave up? concave down?

VII. Optimization problems.

(A) A rectangular box with no top is created out of a rectangular piece of cardboard by cutting equal squares out of the four corners and folding up the sides. If the original piece of cardboard was 20 inches by 15 inches, what is the largest volume possible for the resulting box? (Hint: Let x be the side of the four squares cut out of the corners. The volume is length times width times height.)

(B) A rectangular poster is to be created with 400 square inches of printed material surrounded by 2 inch margins on the top and bottom and the left and right edges. What should the dimensions of the poster be to minimize the total area (printed material plus margins)?

(C) A billboard 20 feet tall is mounted 10 feet above eye level on the wall of a building. How far should a person stand from the wall in order to maximize the angle θ subtended by the billboard at the person's eye. (Hint: Draw a diagram first; see the top diagram in Figure 30 on page 248 of the text if you cannot figure out what this means.)

(D) A window has the shape of a rectangle surmounted by a semicircle (see Figure 10 on page 245 of our textbook if you don't understand what this means). The total perimeter of the window is 600 cm. What should the dimensions be to make the area of the window be as large as possible (so that it will admit the most light possible)?