MATH 133 – Calculus with Fundamentals 1
Discussion Day on Continuity
September 29, 2017

Background

We say $f(x)$ is continuous at $x = c$ if $\lim_{x\to c} f(x) = f(c)$. Intuitively this means that the graph has no hole, jump, or vertical asymptote at $x = c$, although you should be aware that this is far from a complete list of all the possible types of *discontinuities*.

Questions

(1) Consider the function

$$f(x) = \begin{cases} x^2+3 & \text{if } x < 1 \\ -x+10 & \text{if } 1 \le x \le 2 \\ -x^2+6 & \text{if } x > 2, x \ne 3 \\ 10 & \text{if } x = 3. \end{cases}$$

Without using a graphing calculator, generate a plot of the graph $y = f(x)$ that is accurate enough to let you answer the following questions. At each of the given x values, is f continuous? Why or why not? (That is, if f is not continuous there, what part of the definition of continuity fails?) And if f is not continuous there could you redefine the value $f(x)$ to make a new continuous function?

(a) $x = 1$
(b) $x = 2$
(c) $x = 3$.

(2) Which of the following quantities would you expect to be represented by continuous functions of time and which would have one or more discontinuities? Explain.

(a) The velocity of a flying airplane.
(b) The temperature in the middle of this room.
(c) The balance in a bank account.
(d) The population of Worcester.

(3) In 2009, the Federal income tax T on on incomes x up to $\$82,250$ was determined by the formula

$$T(x) = \begin{cases} 0.10x & \text{if } 0 \le x \le 8350 \\ 0.15x - 417.50 & \text{if } 8350 \le x < 33950 \\ 0.25x - 3812.50 & \text{if } 33950 \le x < 82250. \end{cases}$$

Does T have any discontinuities? Why or why not? Might it be advantageous in some situations to earn *less* money?