MATH 136 – Calculus 2
Discussion Day on Volumes by Cavalieri's Principle
September 28, 2016

Background

Recall *Cavalieri's Principle*: If we have a solid object "lined up" along the x-axis as a reference line,

- the solid extends from $x = a$ to $x = b$, and
- the *area of the cross-section* of the solid at a general location x is given by a known function $A(x)$, then

the volume of the solid is computed by the definite integral of the cross-section area function:

$$V = \int_a^b A(x)\ dx.$$

Today, we will practice using this on some interesting examples.

Questions

(1) First a "thought question" (no calculations!) Suppose we have an "oblique" circular cylinder of radius r, where the axis (the "center line") of the cylinder meets the plane of each of the circular cross-sections at an angle $\theta \neq \frac{\pi}{2}$. Draw such a cylinder and determine its volume, if the height (the vertical distance between the top and the bottom) is h Explain how you can tell by using Cavalieri's Principle.

(2) Given: The area enclosed by an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab. Using this fact, find the volume of a right cone of height h whose base is the ellipse $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$. Note: A, B are given values here. The formula for the area enclosed by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ works no matter what a and b are.

(3) A plane inclined at an angle of $\frac{\pi}{4}$ passes through a diameter of the base of a right circular cylinder of radius R. Find the volume of the region within the cylinder and below the plane. There is a picture in problem 20 on page 349 of our textbook that may help you visualize this and find a good way to slice it and apply Cavalieri.

(4) The base of a solid is the triangle with vertices at $(0, 0), (1, 0), (0, 1)$. The slices by planes perpendicular to the x-axis are semicircles with diameters extending from the x-axis up to the point on the line through $(1, 0)$ and $(0, 1)$ with that x-coordinate. What is the volume of this solid?