8 + 4 + 2 = 14 total points

MATH 136 — Problem Set 1B Solutions

5.1/83 8 points distributed as indicated for part credit

(a) With $f(x) = e^x$, $[a,b] = [0,1]$, and N subdivisions we have $\underbrace{\Delta x = \frac{1-0}{N} = \frac{1}{N}, \quad x_j = \frac{j}{N}}_{1}$; $j = 0, 1, \ldots, N$. So

$$L_N = \sum_{j=1}^{N} e^{\frac{j-1}{N}} \cdot \frac{1}{N} \overset{1}{=} \sum_{j=0}^{N-1} e^{j/N} \cdot \frac{1}{N}$$ (note change in index for summation)

$$= \frac{1}{N} \sum_{j=0}^{N-1} e^{j/N} \quad 1$$

(b) Since $e^{j/N} = (e^{1/N})^j$ (rules for exponents), this is a geometric sum and the given formula (8) w/ $r = e^{1/N}$ shows

$$L_N = \frac{1}{N} \cdot \frac{(e^{1/N})^N - 1}{e^{1/N} - 1} \ 1 = \frac{e - 1}{N(e^{1/N} - 1)} \ 1 .$$

(c) The numerator in L_N doesn't depend on N. The denominator $N(e^{1/N} - 1)$ gives an indeterminate form $\infty \cdot 0$ as $N \to \infty$. So applying L'Hopital's Rule:

$$\lim_{N \to \infty} N(e^{1/N} - 1) = \lim_{N \to \infty} \frac{e^{1/N} - 1}{\frac{1}{N}} \ 1 \quad \left(\frac{0}{0}\right) \quad \frac{f(N)}{g(N)}$$

$$= \lim_{N \to \infty} \frac{e^{1/N} \cdot \cancel{\left(-\frac{1}{N^2}\right)}}{\cancel{-\frac{1}{N^2}}} \ 1 \ \text{(chain rule)} \quad \frac{f'(N)}{g'(N)}$$

$$= \lim_{N \to \infty} e^{1/N} = 1. \ 1$$

Hence $\lim_{N \to \infty} L_N = \dfrac{e-1}{1} = \boxed{e-1}$ 1.

5.2/48 4 total

$$\int_0^b x^3\,dx = \lim_{N\to\infty} R_N$$

$$= \lim_{N\to\infty} \sum_{j=1}^{N} \left(\frac{jb}{N}\right)^3 \cdot \frac{b}{N} \quad 2$$

$$= \lim_{N\to\infty} \frac{b^4}{N^4} \sum_{j=1}^{N} j^3$$

$$= \lim_{N\to\infty} \frac{b^4}{N^4}\left[\frac{N^4}{4} + \frac{N^3}{2} + \frac{N^2}{4}\right] \quad 1$$

(formula 5 on p. 265)

$$= \lim_{N\to\infty} \frac{b^4}{4} + \cancel{\frac{b^4}{2N}}^{\,0} + \cancel{\frac{b^4}{4N^2}}^{\,0}$$

$$= \boxed{\frac{b^4}{4}} \quad 1.$$

5.2/50 2 total

$$\int_1^3 x^3\,dx = \overbrace{\int_0^3 x^3\,dx - \int_0^1 x^3\,dx}^{1}$$

$$= \frac{81}{4} - \frac{1}{4} \quad \text{(using 48)}$$

$$= \boxed{20} \quad 1$$

−1 If they just say

$$\int_1^3 x^3\,dx = \frac{x^4}{4}\Big|_1^3 = \frac{81}{4} - \frac{1}{4}$$

and note: we don't know that yet!