10, 6, 4 (Total = 20)

MATH 136, Problem Set 9 'B' Solutions

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(a) $c_1 = 1 + \frac{1}{2} = \frac{3}{2}$

$c_2 = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$ (2) ($\frac{1}{2}$ each-1

$c_3 = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{19}{20}$

$c_4 = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} = \frac{743}{840}$

(b) Consider Riemann sums for $f(x) = \frac{1}{x}$ on $[n, 2n]$ with $\Delta x = 1$. The left-hand sum is

$$\frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n-1} > \int_n^{2n} \frac{1}{x}\,dx \quad \text{since } \frac{1}{x} \text{ is } \underline{\text{decreasing}}$$

$$\therefore \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n-1} + \frac{1}{2n} = c_n > \int_n^{2n} \frac{1}{x}\,dx + \frac{1}{2n}$$

Similarly, the right-hand sum is

$$\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n} < \int_n^{2n} \frac{1}{x}\,dx$$

so $$\frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n} = c_n < \int_n^{2n} \frac{1}{x}\,dx + \frac{1}{n}$$

Hence $$\int_n^{2n} \frac{1}{x}\,dx + \frac{1}{2n} < c_n < \int_n^{2n} \frac{1}{x}\,dx + \frac{1}{n}$$ (4)

(c) $$\int_n^{2n} \frac{1}{x}\,dx = \ln x \Big|_n^{2n} = \ln(2n) - \ln(n) = \ln\left(\frac{2n}{n}\right) = \ln(2)$$ (2)

Hence

$$\lim_{n\to\infty} \left[\int_n^{2n} \frac{1}{x}\,dx + \cancel{\frac{1}{2n}}^{\,0}\right] = \ln(2)$$

$$\lim_{n\to\infty} \left[\int_n^{2n} \frac{1}{x}\,dx + \cancel{\frac{1}{n}}^{\,0}\right] = \ln(2)$$

(2)

By the Squeeze theorem, $\boxed{\lim_{n\to\infty} c_n = \ln(2)}$

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The long term level (just before next dose) is

(a) $$\lim_{n\to\infty}\left[\underset{\uparrow \text{ 1 day before}}{De^{-k}} + \underset{\uparrow \text{ 2 days before}}{De^{-2k}} + \underset{\uparrow \text{ 3 days before} \ldots}{De^{-3k}} + \cdots + De^{-nk}\right]$$

this is a geometric series with $\begin{cases} a = De^{-k} \\ r = e^{-k}\end{cases}$

So the sum is $\boxed{\dfrac{De^{-k}}{1-e^{-k}}}$ (2)

(b) Similar to (a) but with $r = e^{-kt}$ as the doses are taken t days apart: Sum is $R = \dfrac{De^{-kt}}{1-e^{-kt}}$ (2)

(c) When the patient takes the next dose D we want

$D + R \le S$. Now, $D + R = S$ when $S = D + \dfrac{De^{-kt}}{1-e^{-kt}}$

$= \dfrac{D}{1-e^{-kt}}$. So $1 - e^{-kt} = \dfrac{D}{S}$

$$e^{-kt} = \frac{S-D}{S}$$

$$\boxed{t = -\frac{1}{k}\ln\left(\frac{S-D}{S}\right)}$$ (2)

If $t > -\frac{1}{k}\ln\left(\frac{S-D}{S}\right)$, then R is smaller and the patient is even safer.

58. The painted area is given by the geometric series

$\frac{1}{3^2} + 8\cdot\frac{1}{3^4} + 8^2\cdot\frac{1}{3^6} + \cdots$ with $a = \frac{1}{9}$, $r = \frac{8}{9}$. The sum is $\dfrac{1/9}{1-8/9} = 1.$ (!) (4)