8, 4, 4, 2 18 total

MATH 136, Problem Set 8 'B' Solutions

9.3/14 $F(t,y) = t^2 - y$

Euler's method for $\frac{dy}{dt} = t^2 - y$:

$y_0 = 3$ from the initial condition $y(2)=3$

1 (a) $y_1 = 3 + F(2,3)\overset{h}{\Delta t} = 3 + (4-3)(.1) = \underline{3.1}$

1 (b) $y_2 = 3.1 + F(2.1, 3.1)(.1) = 3.1 + ((2.1)^2 - 3.1)(.1) = \underline{3.231}$

4 (c) $y_3 = 3.231 + F(2.2, 3.231)(.1) = 3.231 + ((2.2)^2 - 3.231)(.1) = \underline{3.3919}$

$y_4 = 3.3919 + F(2.3, 3.3919)(.1) = 3.3919 + ((2.3)^2 - 3.3919)(.1) = \underline{3.58171}$

$y_5 = 3.58171 + F(2.4, 3.58171)(.1) = 3.58171 + ((2.4)^2 - 3.58171)(.1) = \underline{3.799539}$

$y_6 = 3.799539 + F(2.5, 3.799539)(.1) = 3.799538 + ((2.5)^2 - 3.799539)(.1)$

$= \underline{4.0445851}$

2 (d) $y(2.2) \doteq y_2 = 3.231$

$y(2.5) \doteq y_5 = 3.799539$

9.4/8

1 (a) $\frac{dy}{dt} = ky(1-y)$

(b) the general solution of the equation in (a) is

$$y = \frac{1}{1 + a e^{-kt}}$$

$y(0) = .1$ so $.1 = \frac{1}{1 + a \cdot 1}$

$\therefore \boxed{a = 9}$ 1

then $y(2) = .4 = \frac{1}{1 + 9e^{-2k}}$ so

$$2.5 = 1 + 9e^{-2k}$$

$$-\frac{1}{2}\ln\left(\frac{1.5}{9}\right) = k$$

so $k \doteq .8959$ 1 (unit 1/day)

(c) $\underset{3/4}{\overset{\shortparallel}{.75}} = \dfrac{1}{1+9e^{-(.8959)t}}$, solve for t

$$\frac{4}{3} = 1 + 9e^{-(.8959)t}$$

$$\frac{1}{3} = 9e^{-.(8959)t}$$

so $t \doteq \dfrac{-1}{.8959}\ln\left(\dfrac{1}{27}\right) \doteq 3.68$ 1

(between 3 and 4 days).

9.5/41 the equation $\dfrac{dy}{dt} = -k(y - a - b\sin wt)$ is first order linear:

$$\frac{dy}{dt} + \underset{P(t)}{\overset{\shortparallel}{k}}\,y = \underbrace{ak + bk\sin(rt)}_{Q(t)}$$

the general solution is derived from $e^{\int P(t)dt} = e^{kt}$

$$y = Ce^{-kt} + e^{-kt}\int e^{kt}\left(ak + bk\sin(rt)\right)dt$$

$$(*) \qquad = Ce^{-kt} + a + bke^{-kt}\int e^{kt}\sin(rt)\,dt$$

To do the remaining integral, use parts (twice)

$$\int e^{kt}\sin(rt)\,dt \qquad \begin{cases} u = e^{kt} & du = ke^{kt}\,dt \\ dv = \sin(rt) & v = -\frac{1}{r}\cos(rt) \end{cases}$$

$$= -\frac{1}{r}e^{kt}\cos(rt) + \frac{k}{r}\int e^{kt}\cos(rt)\,dt$$

parts again

$$u = e^{kt} \qquad du = ke^{kt}\,dt$$

$$dv = \cos(rt) \qquad v = \frac{1}{r}\sin(rt)$$

$$= -\frac{1}{r}e^{kt}\cos(rt) + \frac{k}{r}\left[\frac{1}{r}e^{kt}\sin(rt) - \frac{k}{r}\int e^{kt}\sin(rt)\,dt\right]$$

so

$$\underbrace{\left(1+\frac{k^2}{r^2}\right)}_{=\frac{k^2+r^2}{r^2}}\int e^{kt}\sin(rt)\,dt = e^{kt}\left[-\frac{1}{r}\cos(rt) + \frac{k}{r^2}\sin(rt)\right]$$

and $$\int e^{kt}\sin(rt) = \frac{e^{kt}}{k^2+r^2}\left[-r\cos(rt) + k\sin(rt)\right]$$

Substituting back into (*) above,

(4) $$y = Ce^{-kt} + a + \frac{bk}{k^2+r^2}\left[k\sin(rt) - r\cos(rt)\right].$$

9.5/42 Using the values $k = .1$, $a = 15$, $b = 5$, $r = \pi/12$ in the general solution from 41 we get

$$y(t) = Ce^{-(.1)t} + 15 + \frac{(5)(.1)}{(.1)^2 + (\pi/12)^2}\left[(.1)\sin\left(\frac{\pi}{12}t\right) - \frac{\pi}{12}\cos\left(\frac{\pi t}{12}\right)\right]$$

We want $y(0) = 25$, so

$$25 = C + 15 + \left[\frac{(5)(.1)}{(.1)^2 + (\pi/12)^2}\right]\left((.1)\cdot 0 - \frac{\pi}{12}\cdot 1\right)$$

2 so $C = 10 + \dfrac{5 \cdot \frac{\pi}{12} \cdot (.1)}{(.1)^2 + (\pi/12)^2} \doteq \boxed{11.67}$