7, 6, 2 (w/ 2 Extra Credit), 4 = 19 Total

MATH 136, Problem Set 6, Part 'B' Solutions.

7.5/62 (a) If $Q(x) = (x-a_1)\cdots(x-a_n)$, then by the product rule,

$$Q'(x) = \sum_{j=1}^{n} (x-a_1)\cdots\widehat{(x-a_j)}\cdots(x-a_n)$$

[means: $(x-a_j)$ factor omitted]

So 1 $Q'(a_i) = (a_i-a_1)\cdots(a_i-a_{i-1})(a_i-a_{i+1})\cdots(a_i-a_n)$, since all the other terms in $Q'(x)$ are zero at $x=a_i$.

If

$$\frac{P(x)}{Q(x)} = \frac{A_1}{x-a_1} + \cdots + \frac{A_n}{x-a_n}$$

then we get after multiplying through by $Q(x)$:

(x-an omitted)

$$P(x) = A_1(x-a_2)\cdots(x-a_n) + \cdots + A_n(x-a_1)\cdots(x-a_{n-1})$$

2

(x_1-a_1 omitted)

(the term with A_i has all the factors except $x-a_i$)

So $P(a_i) = A_i(a_i-a_1)\cdots(a_i-a_{i-1})(a_i-a_{i+1})\cdots(a_i-a_n)$

since all the other terms have $x-a_i$ factors, so are 0 at $x=a_i$

$= A_i Q'(a_i)$ from above.

this shows $\boxed{A_i = \dfrac{P(a_i)}{Q'(a_i)}}$ 1

(b) $$\frac{2x^2-1}{(x+1)(x-2)(x-3)} = \frac{A_1}{x+1} + \frac{A_2}{x-2} + \frac{A_3}{x-3}$$

$a_1 = -1$ $\quad P(x) = 2x^2 - 1$

$a_2 = 2$ $\quad Q'(x) = 3x^2 - 8x + 1$

$a_3 = 3$

So $$A_1 = \frac{P(-1)}{Q'(-1)} = \frac{1}{12}$$ 3

$$A_2 = \frac{P(2)}{Q'(2)} = \frac{7}{-3} = -\frac{7}{3}$$

$$A_3 = \frac{P(3)}{Q'(3)} = \frac{17}{4}$$

7.6/48

$$\int e^x \sqrt{e^{2x}-1}\, dx$$

If we let $w = e^x$, $dw = e^x\, dx$, and this becomes

1 $$\int \sqrt{w^2-1}\, dw$$

this could also be combined as $e^x = \sec\theta$

Now let $w = \sec\theta$ $\quad dw = \sec\theta \tan\theta\, d\theta$

$$= \int \underbrace{\sqrt{\sec^2\theta - 1}}_{=\tan\theta} \cdot \sec\theta \cdot \tan\theta\, d\theta$$

1 $$= \int \sec\theta \cdot \tan^2\theta\, d\theta$$

If we do this "by hand" it's a complicated one! If we use parts with $dv = \sec\theta \tan\theta\, d\theta$ $\quad v = \sec\theta$

$u = \tan\theta$ $\quad du = \sec^2\theta\, d\theta$

we get

also OK if they do part of this by consulting a table, but add a note: "see solution"

$$\int \sec\theta \tan^2\theta \, d\theta = \sec\theta\tan\theta - \int \sec^3\theta \, d\theta$$

$$= \sec\theta\tan\theta - \int \underbrace{\sec^2\theta}_{\text{identity}} \cdot \sec\theta \, d\theta$$

$$= \sec\theta\tan\theta - \int (\tan^2\theta + 1) \sec\theta \, d\theta$$

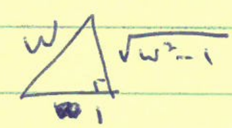
$$= \sec\theta\tan\theta - \int \sec\theta\tan^2\theta \, d\theta - \int \sec\theta \, d\theta$$

so $$2\int \sec\theta\tan^2\theta \, d\theta = \sec\theta\tan\theta - \int \sec\theta \, d\theta$$

$$\int \sec\theta\tan^2\theta \, d\theta = \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\int \sec\theta \, d\theta$$

2 $$= \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$

Convert back: $w = \sec\theta$ so



$$\Rightarrow \tan\theta = \sqrt{w^2-1} \quad 1$$

so we get $$\frac{1}{2} w\sqrt{w^2-1} - \frac{1}{2}\ln|w + \sqrt{w^2-1}| + C$$

$$= \frac{e^x}{2}\sqrt{e^{2x}-1} - \frac{1}{2}\ln|e^x + \sqrt{e^{2x}-1}| + C.$$

1

7.7|84

(a) $V = \int_1^\infty \pi \left(\frac{1}{x}\right)^2 dx = \lim_{R\to\infty} -\frac{\pi}{x}\Big|_1^R = \lim_{R\to\infty} \pi - \frac{\pi}{R} = \boxed{\pi}$ 2

(b) $A = \int_1^\infty 2\pi \frac{1}{x}\sqrt{1 + \frac{1}{x^4}}\, dx$ is given

make this part 2 EXTRA CREDIT

Note that

$$\frac{1}{x}\sqrt{1+\frac{1}{x^4}} \geq \frac{1}{x} \quad \text{for all } x \text{ since } \sqrt{1+\frac{1}{x^4}} \geq 1$$

and $$\int_1^\infty \frac{1}{x}\,dx = \lim_{R\to\infty}\int_1^R \frac{1}{x}\,dx = \lim_{R\to\infty} \ln R = +\infty.$$

so the integral giving A does not converge (See theorem 3 on p. 419)

7.8/13 We want

$$P(0\le r\le a_0) = \int_0^{a_0} 4a_0^{-3} r^2 e^{-2r/a_0}\,dr$$

Let $u = +2r/a_0$, so $du = +2/a_0\,dr$ or $dr = \frac{a_0}{2}du$

$\frac{a_0 u}{2} = r$

$$= \int_{u=0}^{u=2} 4a_0^{-3}\left(\frac{a_0 u}{2}\right)^2 e^{-u}\cdot\frac{a_0}{2}\,du$$

$$= \frac{1}{2}\int_{u=0}^{u=2} u^2 e^{-u}\,du$$ Integrating by parts twice

$$= \frac{1}{2}\left[-u^2e^{-u} - 2ue^{-u} - 2e^{-u}\right]_0^2$$

$$= \frac{1}{2}\left[-10e^{-2}+2\right]$$

$$= \boxed{1-5e^{-2} \doteq .323}$$

4