4, 4, 4, 8 = 20 total

MATH 136, Problem Set 4, 'B'

6.2/20 the cross-section by a plane $\perp$ to the diameter is a triangle 1 (45-45-90) with base and height $\sqrt{r^2-x^2}$ 1

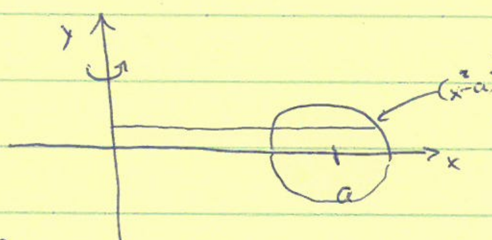
so $A(x) = \frac{1}{2}\left(\sqrt{r^2-x^2}\right)^2 = \frac{1}{2}(r^2-x^2)$ 1 then

$$V = \int_{-r}^{r} A(x)\,dx = \int_{-r}^{r} \tfrac{1}{2}(r^2-x^2)\,dx = \frac{r^2x}{2} - \frac{x^3}{6}\Big|_{-r}^{r} = \frac{r^3}{3}+\frac{r^3}{3}$$

1

$$= \boxed{\frac{2r^3}{3}}$$ 1

6.3/56



$(x-a)^2+y^2=b^2$, $b<a$

so $x = a \pm \sqrt{b^2-y^2}$

the horizontal slices are washers with $r_{in} = a-\sqrt{b^2-y^2}$, $r_{out} = a+\sqrt{b^2-y^2}$ 1

so $V = \int_{-b}^{b} \pi\left(a+\sqrt{b^2-y^2}\right)^2 - \pi\left(a-\sqrt{b^2-y^2}\right)^2 dy$ 1

$= \pi\int_{-b}^{b} (\cancel{a^2} + 2a\sqrt{b^2-y^2} + \cancel{b^2}-\cancel{y^2}) - (\cancel{a^2} - 2a\sqrt{b^2-y^2} + \cancel{b^2}-\cancel{y^2})\,dy$ 1

$= 4\pi a\int_{-b}^{b} \sqrt{b^2-y^2}\,dy$ 1

this integral also computes the area of a semicircle

$= 4\pi a \cdot \frac{1}{2}\pi b^2$ 1

$= \boxed{2\pi^2 a b^2}$.

7.1/84 $\int \underbrace{x^n}_{u} \overbrace{\sin x\,dx}^{dv} = -\underbrace{\overset{u}{x^n}\cos x}_{v} + n\int \overbrace{x^{n-1}}^{du} \overset{v}{\cos x}\,dx$ 2

$\int \underbrace{x^n}_{u} \underbrace{\cos x\,dx}_{dv} = \underbrace{x^n}_{u}\underbrace{\sin x}_{v} - n\int \overbrace{x^{n-1}}^{du} \overbrace{\sin x}^{v}\,dx$ 2

7.1/88

(a) by parts: $\int_0^a f(x)\,dx$; let $\overbrace{u = f(x) \quad dv = dx}^{2}$

$u = f'(x)\,dx \quad V = x$

$$\int_0^a f(x)\,dx = x f(x) \Big|_0^a - \int_0^a x f'(x)\,dx$$

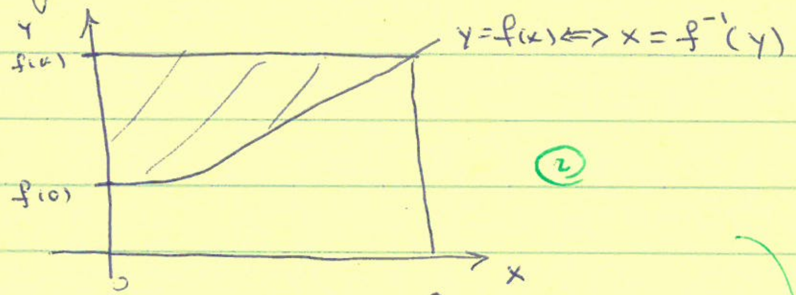
$$= a f(a) - \int_0^a x f'(x)\,dx \quad 2$$

(b) By substitution $u = f(x) \iff x = f^{-1}(u)$

$du = f'(x)\,dx$

$$\int_0^a x \underbrace{f'(x)\,dx}_{du} = \int_{f(0)}^{f(a)} f^{-1}(u)\,du \quad 2$$

this equals the shaded area in



2

$$\therefore \quad a f(a) - \int_0^a f(x)\,dx = \int_{f(0)}^{f(a)} f^{-1}(y)\,dy$$

$$= \int_{f(0)}^{f(a)} f^{-1}(u)\,du$$

$$= \int_0^a x f'(x)\,dx$$

Hence, rearranging, $\int_0^a f(x)\,dx = a f(a) - \int_0^a x f'(x)\,dx$