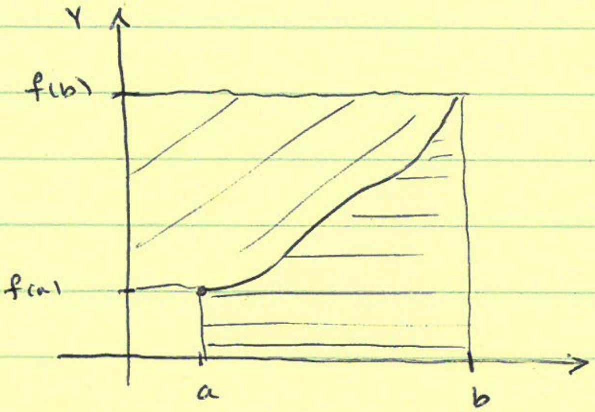
4, 4, 8 = 16 total

MATH 136 Problem Set 3, 'B'

Chapter 5 Review / 118



The term $b \cdot f(b)$ is the area of the rectangle with opposite corners at $(0,0)$ and $(b, f(b))$. (1) Similarly $a \cdot f(a)$ is the area of the rectangle with opposite corners at $(0,0)$, $(a, f(a))$. (1)

Since $f(x) > 0$, $\int_a^b f(x)\,dx$ computes the area of the region shaded $\equiv$. (1)

therefore

$$I = b \cdot f(b) - a \cdot f(a) - \int_a^b f(x)\,dx$$

computes the area of the region shaded ///. (1)

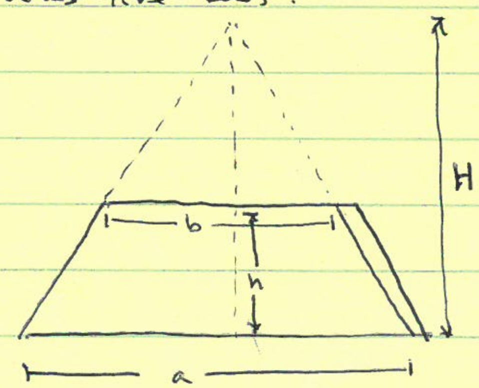
6.1/52 the shaded region has area

$$\int_0^3 f(x) - g(x)\,dx + \int_3^5 f(x)\,dx - \int_5^9 f(x)\,dx$$

(4) (1 point for each integral, 1 point "free")

think $f(x) - 0$ think $+\int_5^9 0 - f(x)\,dx$

6.2/19 (a) the cross-section of the frustum by a plane through the center point of the base and perpendicular to one pair of sides looks like this:



By similar (1) triangles

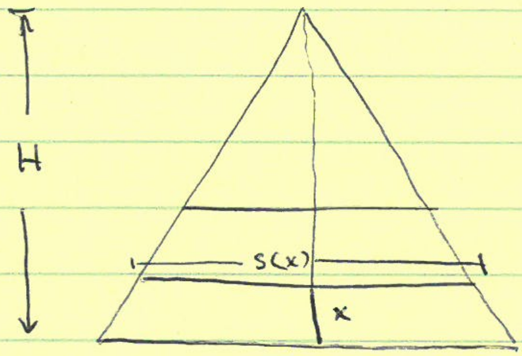
$$\frac{H}{H-h} = \frac{a}{b}$$

so $H = \frac{a}{b}H - \frac{a}{b}h$

$$H\left(1 - \frac{a}{b}\right) = -\frac{a}{b}h$$

$$\boxed{H = \frac{ah}{a-h}}$$ (2)

(b) this also follows from similar triangles



$$\frac{H}{a} = \frac{H-x}{S(x)}$$

so $$S(x) = \frac{a(H-x)}{H}$$

$$= \frac{\cancel{a}\left(\frac{ah}{a-b} - x\right)}{\frac{\cancel{a}h}{a-b}}$$ from (a)

$$= \frac{ah - (a-b)x}{h}$$ 2

$$= \frac{1}{h}\left(a(h-x) + bx\right)$$

as required

(c) the volume of the frustum is the volume of the whole pyramid, minus the volume of the small pyramid with base a square of side b

$$V_{frustum} = \frac{1}{3}a^2 H - \frac{1}{3}b^2(H-h)$$ 2

$$= \frac{1}{3}a^2 \cdot \frac{ah}{a-b} - \frac{1}{3}b^2\left(\frac{ah}{a-b} - h\right)$$ from (a)

$$= \frac{1}{3}\cdot\frac{1}{a-b}\left[a^3 - b^3\right]h$$ 1

$$= \frac{1}{3}\left[a^2 + ab + b^2\right]h.$$ 1 (factor $a^3 - b^3 = (a-b)(a^2+ab+b^2)$)

(Note: Also give full credit if they get the volume of the frustum as $V = \int_0^h \left[\frac{1}{h}\left(a(h-x) + bx\right)\right]^2 dx$