$4+3+8+8+6 = 21$

EC

MATH 136 Problem Set 2 'B' Solutions

I used chain rule; product rule also OK

5.3/

70. $F(x) = \tan^2 x$ has $F'(x) = 2\tan x \sec^2 x$ (1), while $G(x) = \sec^2 x$ has $G'(x) = 2\sec x \cdot \sec x \tan x = 2\tan x \sec^2 x$ (1). Since $F(x) - G(x)$ has derivative 0 for all x, this says $\sec^2 x = \tan^2 x + C$ for some constant C. (1)
The constant $\boxed{C=1}$ from an important trig identity:
Recall $\sin^2 x + \cos^2 x = 1$ for all x. Dividing both sides by $\cos^2 x$ we get

$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

so $$\tan^2 x + 1 = \sec^2 x$$ (1) ← (OK if they just quote this w/o deriving it)

80. $\frac{dV}{dr} = -.06r$ and $V(10) = 0$. We integrate

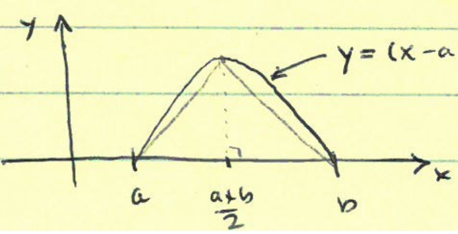
$$V = \int(-.06)r\,dr$$
$$= -(.03)r^2 + C \quad \text{(1)} \quad \text{some } C$$

When $r = 10$, $V = 0$, so

$$0 = V(10) = -(.03)\cdot 10^2 + C = -3 + C$$

$\therefore \boxed{C = 3}$ (1) and $\boxed{V(r) = -(.03)r^2 + 3}$ (1)

5.4 / 62



$y = (x-a)(b-x) = -x^2 + (a+b)x - ab$

the area of the parabolic arch is

$$A_{par} = \int_a^b \overbrace{-x^2 + (a+b)x - ab}^{1}\,dx = \overbrace{\frac{-x^3}{3} + \frac{(a+b)x^2}{2} - abx}^{1}\Bigg|_a^b$$

$$= \left(-\frac{b^3}{3} + \frac{(a+b)b^2}{2} - ab^2\right) - \left(-\frac{a^3}{3} + \frac{(a+b)a^2}{2} - a^2 b\right)$$

$$= b^2\left(-\frac{b}{3} + \frac{a+b}{2} - a\right) - a^2\left(-\frac{a}{3} + \frac{(a+b)}{2} - b\right)$$

$$= \frac{1}{6}\left(b^3 - 3ab^2 + 3a^2 b - a^3\right)$$

$$= \frac{(b-a)^3}{6}.$$

2 any of these forms OK

the base of the triangle is $b-a$, and the height is $\underbrace{-\left(\frac{a+b}{2}\right)^2 + (a+b)\cdot\left(\frac{a+b}{2}\right) - ab = \frac{1}{4}(b-a)^2}_{2}$. Hence

$$A_{tri} = \underbrace{\frac{1}{2}\frac{1}{4}(b-a)^3 = \frac{(b-a)^3}{8}}_{1}. \text{ then } \underbrace{\frac{4}{3}A_{tri} = \frac{4}{3}\cdot\frac{(b-a)^3}{8} = \frac{(b-a)^3}{6}}_{1} = A_{par}.$$

5.4/63 is Extra Credit: add any points here to numerator of score but denominator still

(a) the points are $A = (r, (r-a)(b-r))$ $E = (s, (s-a)(b-s))$

so the slope of the secant line is

$$m_{sec} = \frac{(r-a)(b-r) - (s-a)(b-s)}{r-s} = \frac{-r^2 + (a+b)r + s^2 - (a+b)s}{r-s}$$

$$= \frac{(r-s)\left(-(r+s) + a + b\right)}{r-s}$$

$$= (a+b) - (r+s)$$

2

If $y = -x^2 + (a+b)x - ab$, then $\frac{dy}{dx} = -2x + a + b = m_{tan}$

so $-2x + (a+b) = a+b-(r+s)$ when $\boxed{x = \frac{r+s}{2}}$.

(b) the area of the parallelogram ABDE is

$$(s-r)\left[\left(-\left(\frac{r+s}{2}\right)^2 + \cancel{(a+b)\left(\frac{r+s}{2}\right)} - a\cancel{b}\right) - \frac{(-r^2+\cancel{(a+b)r}-a\cancel{b}) + (-s^2+\cancel{(a+b)s}-a\cancel{b})}{2}\right]$$

$$= (s-r)\left[-\frac{r^2}{4} - \frac{rs}{2} - \frac{s^2}{2} + \frac{r^2}{2} + \frac{s^2}{2}\right]$$

$$= (s-r)\cdot\left(\frac{r^2}{4} - \frac{rs}{2} + \frac{s^2}{4}\right)$$

$$= \frac{(s-r)^3}{4} \quad 2$$

(C) $\therefore$ the triangle ΔACE is one half of this, or $\frac{(s-r)^3}{8}$. 1

(d) the area of the shaded region is

$$\int_r^s -x^2 + (a+b)x - ab \, dx - (s-r)\left[\frac{(-r^2+(a+b)r-ab)+(-s^2+(a+b)s-ab)}{2}\right]$$

$$= -\frac{x^3}{3} + \frac{a+b}{2}\cdot x^2 - abx \Big|_r^s - (s-r)\left[-\frac{r^2}{2} - \frac{s^2}{2} + \frac{(a+b)(r+s)}{2} - ab\right]$$

$$= -\frac{s^3}{3} + \frac{r^3}{3} + \cancel{\left(\frac{a+b}{2}\right)(s^2-r^2)} - \cancel{ab(s-r)} - (s-r)\left[-\frac{r^2}{2} - \frac{s^2}{2} + \cancel{\frac{(a+b)(r+s)}{2}} - \cancel{ab}\right]$$

$$= (s-r)\left[-\frac{s^2}{3} - \frac{sr}{3} - \frac{r^2}{3} + \frac{r^2}{2} + \frac{s^2}{2}\right]$$

$$= (s-r)\left[\frac{s^2}{6} - \frac{sr}{3} + \frac{r^2}{6}\right]$$

2

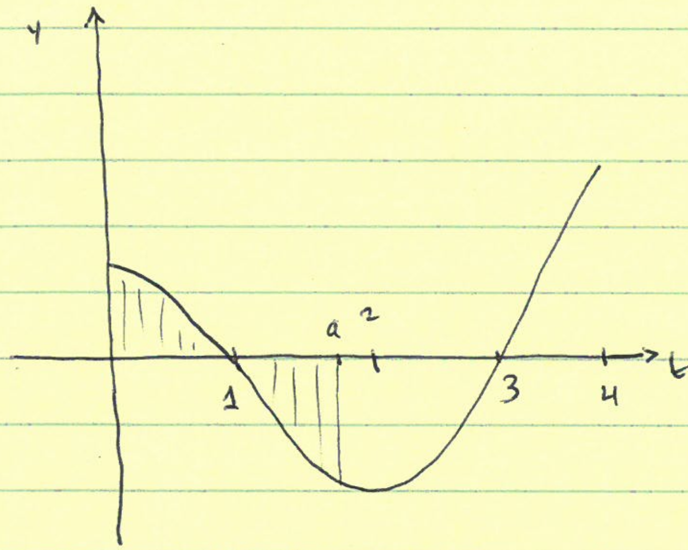
$$= \frac{(s-r)^3}{6}$$

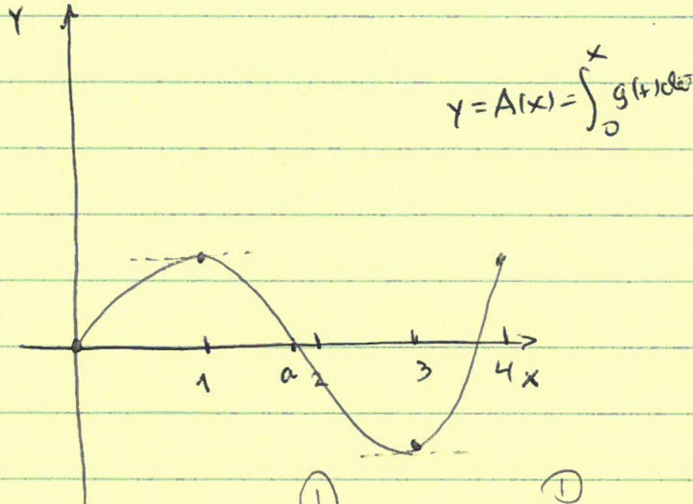
1

As before $\frac{4}{3}\cdot\frac{(s-r)^3}{8} = \frac{(s-r)^3}{6}$. //

5.5/26 $\quad A(x) = \int_0^x g(t)\,dt$, so we think of $A(x)$

as the signed area of the region between the t-axis and $y = g(t)$; $t = 0$, to $t = x$:





$A(a) = 0$ when area above t-axis = area below t-axis

(1)

$A'(x) = 0$ at $x = 1, 3$, ≥ 0 (1)

when $g(x) \geq 0$, ≤ 0 when (1)

[0,1], [3,4]

$g(x) < 0$.

[1,3]

(1) for $A(0) = 0$

(2) for "general shape" but don't get too "picky"