MATH 136 – Calculus 2 Integration By Substitution Practice – Answers September 16, 2016

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$$\int x\sqrt{4x^2 + 16} \, dx$$
. Let $u = 4x^2 + 16$; Answer $= \frac{1}{12}(4x^2 + 16)^{3/2} + C$.
• $\int_0^{\pi} \cos\theta e^{1+\sin\theta} \, d\theta$. Let $u = 1 + \sin\theta$; Answer $= 0$.
• $\int_{\pi/4}^{\pi/2} \cos^3(4\theta) \sin(4\theta) \, d\theta$. Let $u = \cos(4\theta)$; Answer $= 0$.
• $\int \frac{1}{\sqrt{1 - 4x^2}} \, dx$. Let $u = 2x$; Answer $= \frac{1}{2}\sin^{-1}(2x) + C$.

- $\int \frac{x}{\sqrt{1-4x^2}} dx$. Note the difference with the previous one! Let $u = 1 4x^2$, Answer $= -\frac{1}{4}(1-4x^2)^{1/2} + C$.
- $\int \frac{1}{x(\ln(x))^p} dx$. The number p > 0 is a constant; your answer should depend on p and there should be two separate cases: Let $u = \ln(x)$, so the integral is

$$\frac{(\ln(x))^{-p+1}}{-p+1} + C$$

if $p \neq 1$ and

$$\ln(\ln(x)) + C$$

if p = 1.

- $\int x^3 \sec(x^4) \tan(x^4) dx$. Let $y = x^4$. The answer is $\frac{1}{4} \sec(x^4) + C$.
- $\int \frac{\cos(\sqrt{y})}{\sqrt{y}} dy$. Let $u = \sqrt{y}$. The answer is $= 2\sin(\sqrt{y}) + C$.
- $\int x^3 \sqrt{4x^2 + 16} \, dx$. More challenging! Letting $u = 4x^2 + 16$, the $x^3 = \frac{u 16}{4} \cdot \frac{1}{8} du$ and the integral is

$$= \frac{1}{80}(4x^2 + 16)^{5/2} - \frac{1}{3}(4x^2 + 16)^{3/2} + C.$$