Mathematics 136 – Calculus 2
Exam 3 – Review Sheet
November 18, 2016

General Information

As announced in the course syllabus, the third midterm exam of the semester will be given the week after Thanksgiving break. We can either do this in class on Friday, December 2 or in the evening of Thursday, December 1. We can decide this before you leave for Thanksgiving. The format will be similar to that of the first two midterms.

- You may use a non-graphing calculator. Any calculator like the TI-30, which does NOT have graphing capabilities, is OK. (Note: Some of you may have one of these calculators purchased for use in Chemistry courses here. That is also OK.)
- Use of phones, I-pods, I-pads, and any electronic devices other than your calculator *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack (make sure cell phones are turned off).

What will be covered

The exam will cover the material since the last exam (Problem Sets 6, 7, and 8). However some calculations may require you to use integration methods covered on Exam 2 (review integration by u-substitions, parts and trigonometric substitutions in particular!) I understand that this may look like a lot, but to help you organize your studying, keep in mind that *everything here is based on computing integrals by various methods! Review the integration methods first, then the applications.* The new material is:

- Integration by partial fractions (section 7.5): Know the whole process, including preliminary divsion (if necessary), setting up the partial fractions based on the factorization of the denominator, solving for the undetermined coefficients in the partial fractions, then integration of the partial fractions.
- Improper integrals (section 7.7): Know how to recognize when an integral is improper, and how to evaluate these using limits.
- Probability applications (section 7.8): Know how to recognize when a function is a valid probability density (pdf) or compute an undetermined constant to make a function a valid pdf. Know how to compute the probability a random variable has a value in a given interval given the pdf; know how to compute the expected value given the pdf.
- Numerical Integration (section 7.9): In the material from this section, I *will not ask* you to compute approximations using the Midpoint, Trapezoidal, or Simpson's Rules. But I may ask conceptual questions about the various error formulas and bounds, when the Midpoint and Trapezoidal Rules give over- or under-estimates, and what we saw in the computer lab about Simpson's Rule (in particular why that method is expected to yield a smaller error).

- Differential equations, separation of variables, solutions, etc. (section 9.1)
- Models involving $y' = k(y - b)$ (section 9.2): Newton's Law of Heating/Cooling, etc. "word problems" based on that.
- Section 9.3: I *will not ask* you to draw slope fields or carry out Euler's Method calculations "by hand" to derive approximate values of a solution of a differential equation. But you should understand how solutions of a first-order differential equation relate to the slope field. This is another place where I might ask a conceptual question.
- Logistic equations (section 9.4): Know how to derive the general solution of $y' = ky(1 - y/A)$ and how to use those solutions to solve initial value problems.
- First order linear equations (section 9.5): Know how to determine the integrating factor ($e^{\int P(x)\,dx}$) for the equation $y' + P(x)y = Q(x)$ and what the general solution looks like.

Review problems

There is a selection of review problems posted on WebAssign as an optional assignment. I have increased the number of tries on each problem to the maximum possible – 100.

Sample problems

Note: The actual exam will be considerably shorter than the following list of questions and it might not contain questions of all of these types. The purpose here is just to give an idea of the range of different topics that will be covered and how questions might be posed.

I. *Partial Fractions.*

(A) Do you need partial fractions to integrate

$$\int \frac{t^2 + 1}{t^3 + 3t + 3}\,dt?$$

Explain, and compute the integral with a simpler method.

(B) Apply partial fraction decomposition to compute

$$\int \frac{x^3 + 1}{x(x-1)(x+2)}\,dx$$

(Hint: you need to divide through first; do you see why?)

(C) What would the partial fractions look like to integrate

$$\int \frac{1}{x(x^2+4)^2}\,dx?$$

(You don't need to solve for the coefficients, just set up the proper partial fractions.)

(D) Suppose the partial fraction decomposition of a rational function is

$$\frac{1}{81x} + \frac{-\frac{x}{9}+1}{(x^2+9)^2} - \frac{\frac{x}{81}}{x^2+9}$$

What is the integral? (Hint: There's one part of this for which you need a trigonometric substitution.)

II. For each of the following integrals, say why the integral is improper, determine if the integral converges, and if so, find its value.

A) $\displaystyle\int_1^\infty \frac{1}{\sqrt[5]{x}}\,dx$

B) $\displaystyle\int_0^2 \frac{dx}{x^2-7x+6}$

C) $\int_0^\infty xe^{-3x}\,dx$

D) For which values of a is $\int_0^\infty e^{ax}\sin(x)\,dx$ convergent? Evaluate the integral for those a.

III. The time t (in minutes) between two successive incoming calls at a phone center is a random variable with a pdf of the form $f(t) = \frac{1}{r}e^{-t/r}$ if $t \geq 0$ and 0 otherwise.

(A) Suppose that the probability that two successive calls are more than one minute apart is .3. What is the value of r?

(B) Using the value of r you determined in part A, find the probability that the two successive calls are at least 5 minutes apart.

IV. *Numerical Integration.*

(A) If you apply the Trapezoidal Rule and the Midpoint Riemann Sum Rule with $N = 30$ to approximate $\int_1^8 x^2e^x\,dx$, one will overestimate the actual value and one will underestimate the actual value. Which is which? Also, how would we expect the absolute values of the errors for these methods to compare?

(B) How big would you need to take N to get a Midpoint Riemann Sum approximation to $\int_0^1 \sin(x)\,dx$ accurate to 5 decimal places. (Hint: You want the error bound $< .000005$.)

V. *Differential Equations.*

(A) Verify that for any constant C, the function $y = \sqrt{C+x^2}$ is a solution of the differential equation $y\dfrac{dy}{dx} - x = 0$. Which of these solutions also satisfies the initial condition $y(0) = 5$?

(B) For which real a is $y = \sin(ax)$ a solution of the differential equation $\frac{d^4y}{dx^4} - 16y = 0$?

(C) Find the general solution of

$$\sqrt{1-x^2}\frac{dy}{dx} = y^2 + 2y$$

(C) Find the general solution of

$$\frac{dy}{dx} = xe^{x+y}$$

(D) A baked potato is taken out of the oven at a temperature of 140° C and left to cool on a counter in a room maintained at 20° C. After 2 minutes the temperature of the potato has decreased to 100° C. How long will it take for the temperature to reach 80° C?

(E) Sunset Lake is stocked with 2000 rainbow trout. After 1 year the trout population has grown to 4500. Assuming logistic growth with a carrying capacity of 10000 trout, find the growth constant k and determine when the population will reach 5000.

(F) Find the general solution of

$$\frac{dy}{dx} + xy = x$$