MATH 136 – Calculus 2
Worksheet on Antiderivatives
September 7, 2016

Background

As we have just seen, the *indefinite integral* (note – no limits of integration on the integral sign here; that is the "tip-off" that we're not talking about a definite integral, e.g. for an area):

$$F(x) = \int f(x)\ dx$$

is shorthand notation for *the general antiderivative* – that means the most general function such that $F'(x) = f(x)$. For instance as we know the antiderivative Power Rule:

$$\int x^n\ dx = \begin{cases} \dfrac{x^{n+1}}{n+1} + C & \text{if } n \neq -1, \\ \ln|x| + C & \text{if } n = -1. \end{cases}$$

We want to review some material from differential calculus by "thinking backwards" for the rest of the time today. Find the following anti

Questions

1) Find the following antiderivatives (indefinite integrals):

   (a) $\displaystyle\int \sqrt{x} + \frac{1}{\sqrt[3]{x}} + \frac{4}{x^3}\ dx$ (Hint: Write as powers and use the Power Rule above)

   (b) $\displaystyle\int \cos(x/2)\ dx$ ("guess and check")

   (c) $\displaystyle\int e^{5x}\ dx$ ("guess and check")

   (d) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\ dx$ (Hint: What function has a derivative that looks like this? It's not the inverse tangent that we discussed.)

2) How do we know that *every* antiderivative of a function like $f(x) = x^2$ looks like $F(x) = \frac{x^3}{3} + C$? They certainly all "work," but could there be *others too*?

   (a) Suppose you have the antiderivative $F_1(x) = \frac{x^3}{3}$ and $F_2(x)$ is *any other antiderivative* of x^2. What do you get if you take the derivative of the difference $F_2(x) - F_1(x)$?

   (b) What does that say about the difference $F_2(x) - F_1(x)$?

   (c) So what does $F_2(x)$ look like?