Mathematics 136, section 2 – Calculus 2
Information on Final Examination
December 5, 2016

General Information

- The final examination for this class will be given during the scheduled period 11:30am to 2:00pm on Tuesday, December 13.
- Like the midterms, this one will be given *in our regular classroom*, Swords 321.
- The final will be similar in format to the midterm exams but perhaps two times as long. I expect that if you are well prepared and you work steadily, then you should be able to finish the exam in about 1.75 hours. However, you will have the full 2.5 hour period to work on the exam if you need that much time.
- As on the midterms, you may use a calculator, but no graphing features.
- No cell-phones, computers, or other electronic devices beyond a basic calculator may be used during the exam. Please do not bring them with you; they will be subject to confiscation for the period of the exam if you use them.
- I'm happy to try to find a time for a pre-final review session. We can discuss this in class on Wednesday, December 7 or Friday, December 9.
- I will also be available during regular office hours during the week of December 5 and the following time on Monday, December 12 for help as you prepare:
- Monday, December 12, 8:00am - 12:00noon
- Tuesday, December 13, 8:00am - 11:00am

What Will Be Covered

- This will be a *comprehensive* final – it will cover all the topics we have studied this semester, with about 30% each devoted to the material on each of the midterm exams, plus about 10% on the material from Problem Set 9 (sequences and series).
- See the review sheets for the three midterms for a detailed breakdown of the topics we studied earlier. Those review sheets are now reposted on the course homepage if you need another copy of any of them.

Philosophical Comments and Suggestions on How to Prepare

- The reason we give final exams in almost all mathematics classes is to encourage students to "put whole courses together" in their minds. Also, preparing for the final should help to make the ideas "stick" so you will have the material at your disposal to use in later courses. *This is especially important if you are preparing to continue to MATH 241 or other advanced mathematics courses – everything is based on material from this semester's course and it will be difficult or impossible to do well in that course unless you have the material from this one under good control.*

- It may not be necessary to say this, but here goes anyway: *You should take this exam seriously* – it is worth 25% of your course average and it can pull your course grade up *or down* depending on how you do.

- Get started reviewing early and do some work on this *every day* between now and the date of the final. Don't try to "cram" at the end. There's too much stuff that you need to know to approach preparing that way!

- Review videos and your class notes in addition to the text, especially for topics where you lost points on the midterms. There are a lot of worked-out examples and discussions of all of the topics we have covered there.

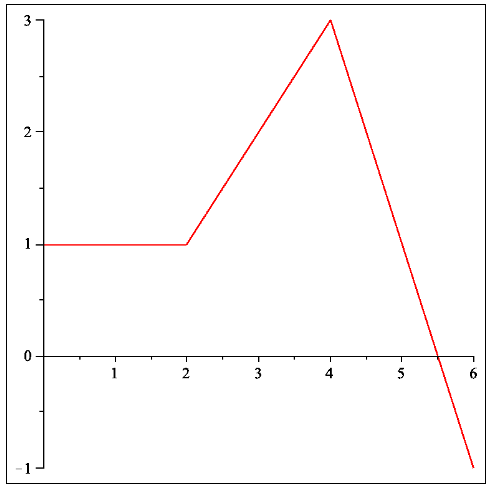
- Look over the midterm exams with the solutions. Go over your corrected problem sets. If there were questions where you lost a lot of points, be sure you understand why what you did was not correct, and how to solve those questions.

- Be sure you actually do enough practice problems so that you have the facility to solve exam-type questions in a limited amount of time. *Even if you have saved solutions for practice problems from the midterms*, it is going to be much more beneficial to do practice problems starting "from scratch" rather than just reading old solutions. Remember, the goal of the course is to get you to be able to develop solutions to these problems yourselves, not just to understand solutions that someone else (that includes you, one or more months ago!) has written down. Another analogy – as most of you know from your study of languages, it's much easier to understand another language passively than it is to actually use a language actively yourself (for instance, to form your own complete, grammatically correct sentences). The goal of this course is to make you reasonably proficient "calculus speakers" and there's no substitute for active practice on those skills.

- The following is (a slightly edited version of) the final I gave in Calculus 2 in spring 2014. It's a good guide for what our exam will look like, but of course, different topics, different types of questions, etc. might also appear. *For instance, you should be prepared to solve separable or first order linear differential equations as well.*

I.

(A) Let $f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 2 \\ x - 1 & \text{if } 2 < x \le 4 \\ 11 - 2x & \text{if } 4 < x \le 6 \end{cases}$. whose graph is shown here:



Let $F(x) = \int_0^x f(t)\,dt$. Complete the following table of values for $F(x)$:

x	0	1	2	3	4	5	6
$F(x)$	0						

(B) (10) Compute the derivative of the function $g(x) = \displaystyle\int_0^{2x} \frac{\cos(t)}{t^2}\,dt$.

II. Compute the following integrals.

(A) $\displaystyle\int \frac{x^4 - 3\pi^3 + \sqrt{x}}{x^{2/3}}\,dx$

(B) $\displaystyle\int x^3 e^{x^2}\,dx$ (use integration by parts)

(C) $\displaystyle\int \frac{\csc^2(5x)\,dx}{\cot(5x) + 7}$

(D) $\displaystyle\int_1^e x^5 \ln(x)\,dx.$

(E) $\displaystyle\int \frac{1}{\sqrt{16 + x^2}}\,dx$

(F) $\displaystyle\int \frac{1}{(x^2 + 1)(x + 1)}\,dx$

III.

(A) The integral

$$\int_1^3 \sqrt{1+9x^4}\, dx$$

would compute the length of the curve $y = x^3$ from $x = 1$ to $x = 3$. Use a midpoint Riemann sum with $n = 4$ to approximate its value.

(B) Given: The graph of the function in the arclength integral is concave up on the interval $[1, 3]$. Check the appropriate box:

The midpoint approximation is a overestimate □/underestimate □.

IV. A region R in the plane is bounded by the graphs $y = 9 - x^2$, $y = 2x$, $x = 0$ and $x = 1$.

(A) Compute the area of the region R.

(B) Compute the volume of the solid obtained by rotating R about the x-axis.

(C) Set up integral(s) to compute the volume of the solid obtained by rotating R about the y-axis. *You do not need to compute the value.*

V. The daily solar radiation x per square meter (in hundreds of calories) in Florida in October has a probability density function $f(x) = c(x-2)(6-x)$ if $2 \leq x \leq 6$, and zero otherwise. Find the value of c, and then compute the probability that the daily solar radiation is > 4 hundred calories per square meter.

VI. An avian flu epidemic has broken out in Birdsburgh, a large city with total population 10 million. Let N be the number, in millions, of people who have been infected, as a function of time t in weeks. The Birdsburgh Public Health Department determines that the rate of change of N is proportional to the product of the number of infected people (N) and the number of people not yet infected.

(A) Write the statement above as a differential equation, calling the constant of proportionality k.

(B) The function $N(t) = \frac{10}{1+9999e^{-t}}$ should be a solution of your differential equation from part A. What is the value of k?

(C) If the epidemic proceeds according the function given in part B, how many weeks will pass before the number of infected people reaches 1 million?

VII.

(A) Does the infinite series $\sum_{n=0}^{\infty} \frac{1}{4^n}$ converge? If so, what is the sum?

(B) Use the Integral Test to determine whether the infinite series $\sum_{n=3}^{\infty} \frac{1}{n(\ln(n))^3}$ converges.

(C) Use the Ratio Test, then consider the endpoints separately, to determine the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{n^2}$.