Mathematics 136 – Calculus 2
Exam 2 – Review Sheet
October 10, 2016

General Information

As announced in the course syllabus, the second midterm exam of the semester will be given Friday, October 28. The format will be similar to that of the first midterm.

- If you want to use a calculator, you will be permitted to bring a TI-30 or similar *non-graphing* calculator. (Note: Some of you may have one of these calculators purchased for use in Chemistry courses here. That is also OK.)
- Use of cell phones, I-pods, I-pads, tablets, and any other electronic device besides a basic calculator *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack (and make sure cell phones are turned off).

What will be covered

The exam will cover the material since the last exam (Problem Sets 3, 4 and 5), namely the following material from sections 6.1, 6.2, 6.3, 7.1, 7.2, 7.3 of Rogawski/Adams:

1. Area between curves
2. Applications of integration and setting up integral formulas: average value of a function over an interval, mass of a rod with known density, two-dimensional circular problems with radially symmetric density, volumes of solids with known cross-sections
3. Volumes of solids of revolution (disk and washer cross-sections)
4. Integration by parts
5. Trigonometric integrals
6. Integration by trigonometric substitution

Important Note: Many of the problems on this exam will require you to set up and compute an integral to find the quantity that is asked for. In addition to knowing how to set up the required integral, the methods of integration tested on the first exam (i.e. basic rules, u-substitution) and the new methods on this exam might be required to evaluate the integral. In other words, this exam is *in effect a cumulative exam on the material since the start of the semester*. Especially there were things you had not mastered on the first exam, you will probably want to begin your review for this exam by going back and looking at the material from sections Chapter 5 in Rogawski/Adams.

There will be a review for the exam in class on Wednesday, October 26.

Review Problems

Chapter 6 review problems: 1 – 45. (Note: A few of these involve functions we have not discussed. For instance, in 12 and 21, $\cosh(x) = \frac{e^x + e^{-x}}{2}$.)

Chapter 7 review problems: 3,4,5,7,8,10,11,13,14,15,16,19,21,23,27

Sample Exam Questions

Disclaimer: The actual exam questions may be organized differently and ask questions in different ways. This list is also quite a bit longer than the actual exam will be (to give you some idea of the range of different questions that might be asked).

I. Compute each of the integrals below using some combination of basic rules, substitution, integration by parts. You must show all work for full credit.

A)
$$\int \frac{e^{\tan^{-1}(x)}}{1+x^2}\, dx$$

B)
$$\int e^x \sin(2x)\, dx$$

C)
$$\int \sin^3(x)\cos^4(x)\, dx$$

D)
$$\int_0^1 \tan^{-1}(x)\, dx$$

E) Use integration by parts to show this reduction formula: If n is a positive integer, then
$$\int x^n e^{ax}\, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\, dx$$

F) Apply the result from part (E) (repeatedly) to compute $\int x^4 e^{-2x}\, dx$.

II.

(A) Verify that $\int \csc\theta\, d\theta = \ln|\csc\theta - \cot\theta| + C$ by differentiating.

(B) Which trigonometric substitution would you apply to compute $\int \frac{1}{u\sqrt{a^2-u^2}}\, du$? What trigonometric integral do you get after making the substitution? Complete the derivation of the integral.

III.

(A) Let R be the region in the plane bounded by $y = 3 - x^2$ and the x-axis.

(1) Sketch the region R.

(2) Find the area of R.

(3) What is the average value of $f(x) = 3 - x^2$ on the interval where it takes positive values?

(4) Let R_1 be the region above the line $y = \overline{y}$ (the average value from (3)) and below $y = 3 - x^2$. Let R_2 be the region above the graph $y = 3 - x^2$ and below $y = \overline{y}$ over the interval where $3 - x^2 \geq 0$. How are the areas of R_1 and R_2 related. (Try to answer this without calculating!)

(5) Find the volume of the solid generated by rotating R about the x-axis.

(B) Let R be the region in the plane bounded by $y = 3x$ and $y = x^2$.

(1) Sketch the region R.

(2) Find the area of R.

(3) Find the volume of the solid generated by rotating R about the x-axis.

(4) Find the volume of the solid generated by rotating R about the y-axis.

(C) Let R be the region in the plane bounded by $y = \cos(\pi x)$, $y = 1/2$, $x = -1/3$ and $x = 1/3$.

(1) Sketch the region R.

(2) Find the area of R.

(3) Find the volume of the solid generated by rotating R about the x-axis.

(4) Find the volume of the solid generated by rotating R about the line $y = -1$.

IV. The height of a monument is 20m. The horizontal cross-section of the monument at x meters from the top is an isosceles right triangle with legs $x/4$ meters. Is the given information enough to find the volume of the monument? If so, find the volume. If not, say why not.

V. A solid paperweight has a circular base of radius 4 cm. The cross-sections of the paperweight by planes perpendicular to one diameter of the base are equilateral triangles. Find the volume of the paperweight.

VI. A *45 rpm single* record was a vinyl disk 7 inches in diameter, with a large central hole 1 inch in diameter. A ring-shaped paper label 1 inch wide was usually glued to the vinyl surface outside the hole, but inside the grooves where the sound was recorded. The vinyl making up the disk, plus the paper of the label, had mass density .1 ounce per square inch at all points outside the hole in the central region. But the density then decreased linearly from .1 to .07 ounces per square inch at the outer edge. The density was the same at all points in the region the same distance away from the center of the hole. Set up an integral

or integrals to compute the total mass of a 45 rpm single record (including the label) and compute the total mass.