MATH 135 – Calculus 1
Answers for Sample Questions for Exam 1
September 19, 2016

I. Express the set of x satisfying $|2x - 5| > 1$ as an interval or union of intervals. *Answer:* $(-\infty, 2) \cup (3, \infty)$ (that is, all $x < 2$, together with all $x > 3$).

II. The following table contains values for three different functions: $f(x), g(x), h(x)$.

x	0	0.1	0.2	0.3	0.4
$f(x)$	-4.2	-5.9	-7.6	-9.3	-11.0
$g(x)$	10	20	40	80	160
$h(x)$	4	2.3	1.5	2.1	6.1

A) One of these is a linear function. Explain how you can tell which one it is, and give a formula for it.

Answer: $f(x)$ is the linear one, since each change of .1 in x changes $f(x)$ by -1.7. The formula is $f(x) = -17x - 4.2$

B) One of these functions is *neither linear nor exponential.* Explain which one that is and why.

Answer: Exponential and linear functions are either increasing for all x or decreasing for all x. That is not true for $h(x)$.

C) Give a possible formula for $g(x)$. (Hint: the values are doubling every time x increases by .1.)

Answer: $g(x) = 102^{t/.1} \doteq 10(1024)^t$

III.

A) Complete the square in the quadratic function $f(x) = -3x^2 + 12x + 21$.

Answer: $f(x) = -3((x - 2)^2 - 11) = 33 - 3(x - 2)^2$

B) What is the maximum value attained by the function $f(x)$, and for which x is the maximum achieved?

Answer: Maximum is 33, attained when $x = 2$.

C) Where does the graph $y = f(x)$ cross the x-axis?

Answer: $x = \frac{-12 \pm \sqrt{144+252}}{-6} = 2 \pm \sqrt{11} \doteq -1.317, 5.317$.

D) Sketch the graph $y = -3x^2 + 12x + 21$ for x in $[-4, 4]$ and showing correct scales on both the x- and y-axes.

Answer: The graph is a parabola opening down from the vertex $(2, 33)$ like this:

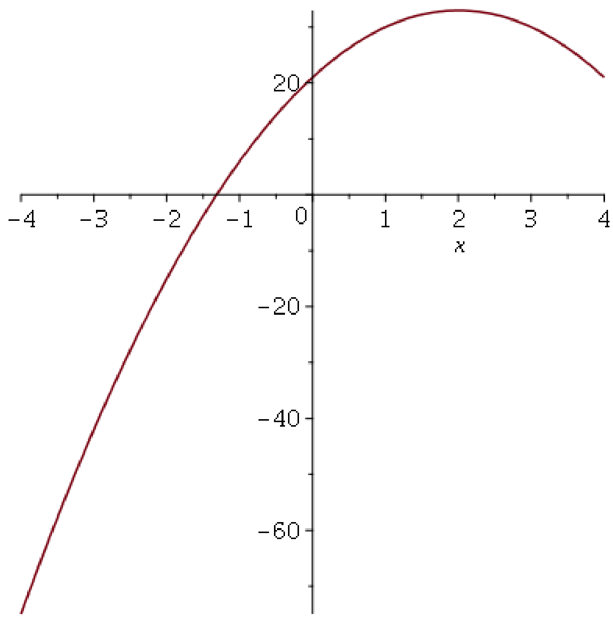

Figure 1: Figure for Question III, part D

IV. You start at $x = 0$ at time $t = 0$ (hours) and drive along the x-axis (x values in miles) at 40 miles an hour for 2 hours. At $t = 2$ you stop for one hour. Then starting at $t = 3$, you retrace your earlier path and return to your starting position at 80 miles per hour.

A) Sketch the graph of your position as a function of time.

Answer:

B) Give (piecewise) formulas for your function on the appropriate t-intervals.

Answer:

$$x(t) = \begin{cases} 40 * t & \text{if } 0 \leq t \leq 2 \\ 80 & \text{if } 2 < t \leq 3 \\ 80 - 80(t-3) & \text{if} 3t \leq 4. \end{cases}$$

V.

A) Express the domain of the function $f(x) = \frac{x}{x^2-1}$ as a union of intervals.

Answer: It is all $x \neq -1, 1$, so $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

B) The figure for this problem shows the graph $y = \frac{x}{x^2-1}$. Based on this, what can you say about the range of $f(x)$?

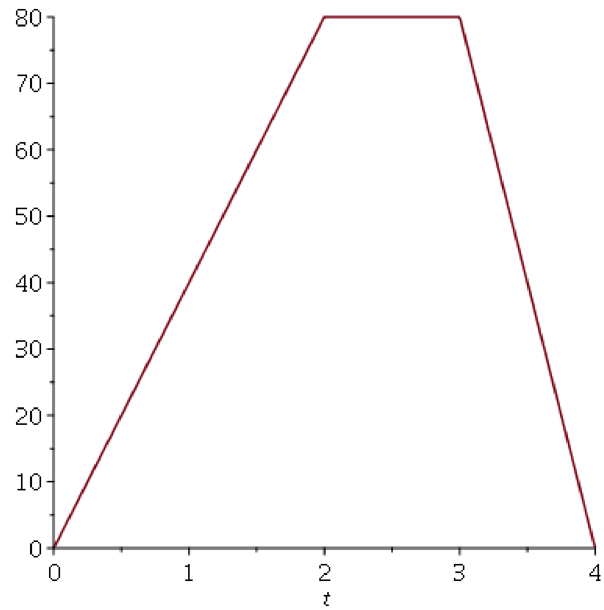

Figure 2: Figure for Question IV,A

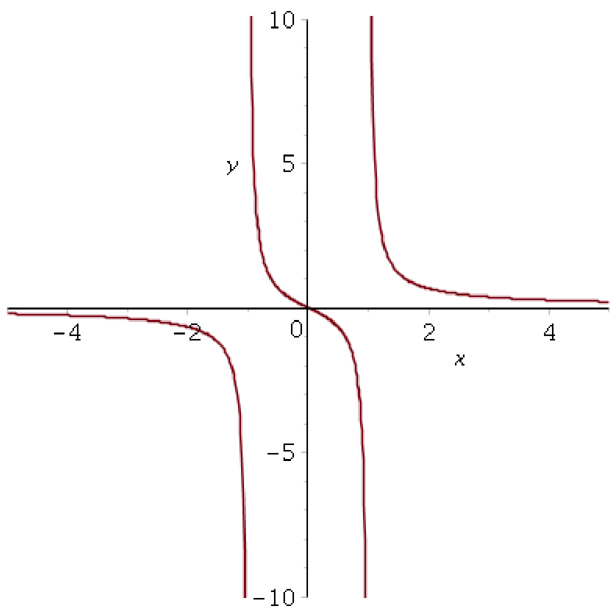

Figure 3: Figure for Question V

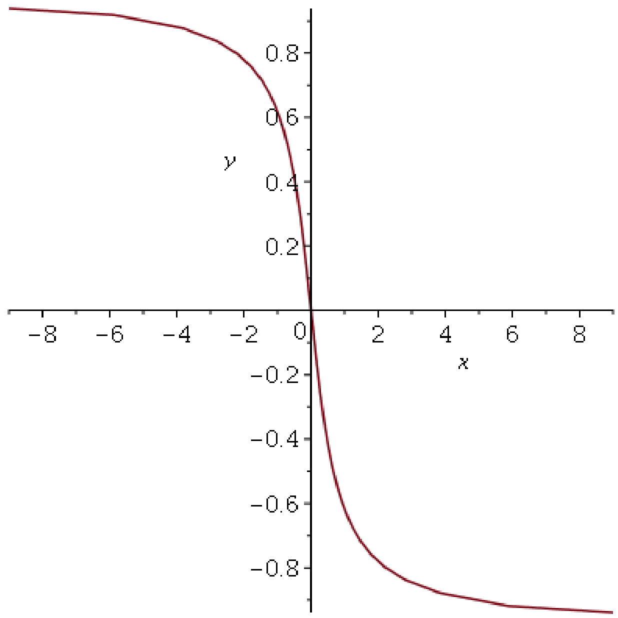

Figure 4: Figure for Question V,D

Answer: Seems to be all real numbers: $\mathbb{R}$, or $(-\infty, \infty)$

C) Explain why $f(x)$ (on its default domain) *fails* to have an inverse function.

Answer: The graph does not pass the horizontal line test, so $f(x)$ is not one-to-one.

D) Give a restricted domain on which $f(x)$ *does* have an inverse function, and sketch the graph of the inverse.

Answer: The interval of x-values $(-1, 1)$ is one such. (The intervals $(1, \infty)$ and $(-\infty, -1)$ would be others.)

VI.

A) Sketch the graph $y = 3\sin\left(\frac{x}{2}\right) + 2$ for $0 \leq x \leq 8\pi$.

B) What are the *amplitude* and *period* of this sinusoidal function?

Answer: Amplitude $= 3$, period $= 4\pi$.

C) What would change in your answer to B) if the formula was $y = \frac{1}{3}\sin(2x) + 2$?

Answer: The amplitude would change to $\frac{1}{3}$ and the period would change to π:

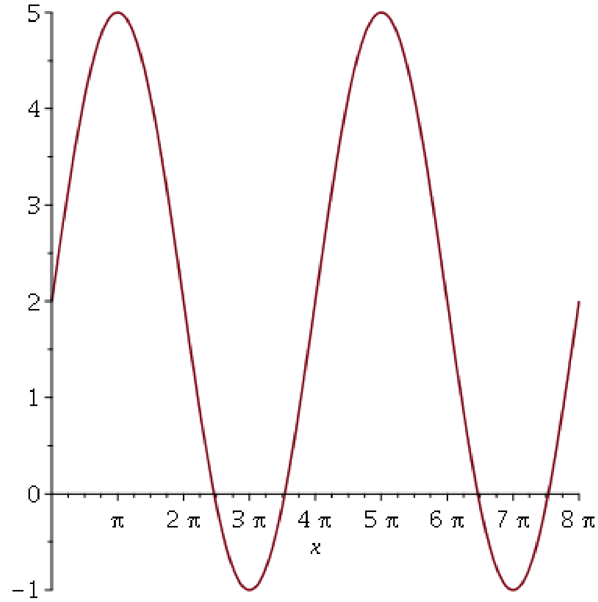

Figure 5: Figure for Question VI, A

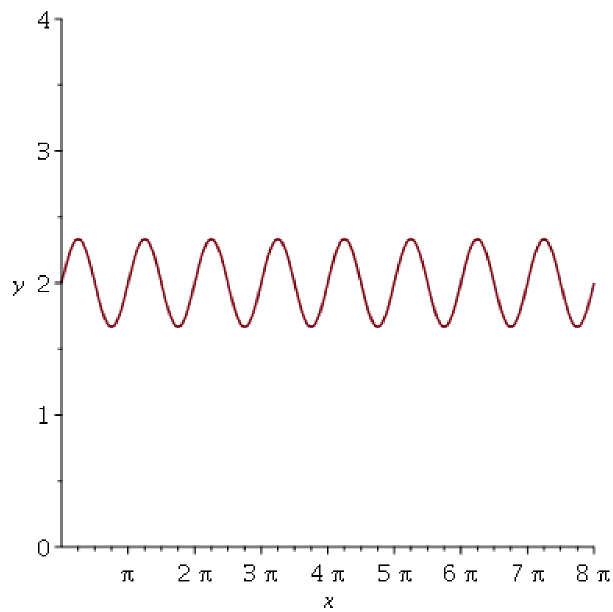

Figure 6: Figure for Question VI, C

VII.

A) Simplify: $\log_3(27) + \ln(e^{-3})$.

Answer: 0

B) Solve for x: $2^{x+3} = 3^{x/2}$.

Answer: $x = \frac{6\ln(2)}{\ln(3)-2\ln(2)}$.

C) The population of a city (in millions) at time t (years) is $P(t) = 2.4e^{0.06t}$. What is the population at $t = 0$? When will the population reach 4 million?

Answer: Population at time $t = 0$ is $P(0) = 2.4$ million. The population reaches 4 million at $t = \frac{\ln(4/2.4)}{.06} \doteq 8.5$ years.

D) (Continuation of C) How long will it take for the population to reach double the number at $t = 0$?

Answer: $t \doteq 11.6$ years.