MATH 135 – Calculus 1
Discussion Day – Related Rates Problems
November 9 and 11, 2016

Background

As explained in the video for today, related rates problems involve situations where two or more quantities are changing with time, but where we know some relation or equation that holds for all times. By differentiating the relation with respect to time, we can get information about unknown rates of change if we know other rates of change. Our textbook presents a three-step process for solving these:

1. Assign names to the quantities (variables) involved and restate the problem in terms of those variables, clearly distinguishing between quantities and rates of change. Drawing a diagram is also often very useful if the problem does not supply one.

2. Find an equation relating the variables and differentiate with respect to t. (Usually, there will not be any explicit t's in your equation, so this will involve something like *implicit differentiation.*)

3. Use the given data and find the unknown rate (derivative).

If you have not viewed the video for today or read section 3.10 in the textbook yet, I highly recommend you do one or both of those now to see some worked examples of these problems.

Questions

(1) A bicyclist moves counterclockwise around an elliptical track in the shape of the curve with equation:

$$\frac{x^2}{1600} + \frac{y^2}{900} = 1, \tag{1}$$

where x and y are distances in meters.

(a) In which of the four quadrants of this xy-coordinate system is $\frac{dx}{dt} > 0$? In which quadrants is $\frac{dy}{dt} < 0$? Explain. Recall that the cyclist is moving counterclockwise around the ellipse.

(b) Find a relation between $\frac{dx}{dt}$ and $\frac{dy}{dt}$ that holds at all times. (Hint: Differentiate (1) with respect to t.)

(c) When the bicyclist moves through the point $(x, y) = (24, 24)$, the y-coordinate of her position is increasing at 3 meters per second. How fast is the x-coordinate of her position changing?

(2) Sand is being poured onto a sand pile at a constant rate of 4 cubic centimeters per second. The pile has the shape of a right circular cone with $h = \frac{r}{2}$ at all times, where h is the height, and r is the base radius. How fast is the height changing when the height is 10 centimeters?

And how fast is the circumference of the base changing at that time? (Note: The volume of a cone is $V = \frac{\pi r^2 h}{3}$.)

(3) A radio-controlled drone is tracked by an observer stationed at ground level standing still exactly 40 meters from the point where the drone took off. The drone ascends along a vertical path perpendicular to the ground.

(a) Suppose the height of the drone is increasing at 10 meters per second. How fast is the angle between the horizontal and the direct line of sight to the drone changing when the angle is $\frac{\pi}{4}$?

(b) If the distance from the observer to the drone is changing at 8 meters per second when the height of the drone is 100 meters, how fast is the drone ascending? (Note: the parts here are separate problems; don't use values from one in the other!)

(4) A laser pointer is placed on a platform that rotates at a constant rate of 20 revolutions per minute. (How many radians per minute is that?) The beam hits a wall 8 meters away from the platform, producing a dot of light that moves horizontally along the wall. Let θ be the angle between the beam and the line through the platform perpendicular to the wall. How fast is the dot of light moving when $\theta = \frac{\pi}{4}$? (Hint: See Figure 10 on page 187 of the text.)

Assignment

One writeup of solutions to these problems from each group, due on Friday, November 11.