MATH 135 – Calculus 1
Discussion Day – "Derivative Practice"
October 24, 2015

*Background*

We have now seen the sum and product rules for derivatives. The goals for today are:

(1) To introduce another rule called the *quotient rule*.

(2) To practice using these rules, and

(3) To think about some of the information about a function that we can get from the derivative.

The quotient rule for derivatives says: If $f, g$ are differentiable and $g(x) \neq 0$ then $f/g$ is differentiable at $x$ and

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

("bottom times derivative of top minus top times derivative of bottom, all over bottom squared"). For instance

$$\frac{d}{dx}\left(\frac{x^2+3}{x^4+x}\right) = \frac{(x^4+x)(2x) - (x^2+3)(4x^3)}{(x^4+x)^2} = \frac{-2x^5 - 4x^3 + 2x^2}{(x^4+x)^2}$$

(This could be simplified even farther by factoring $x^2$ out of the numerator and denominator. But notice that $x = 0$ is not in the domain of the original function, so it's not in the domain of the derivative either. Even though you could cancel $x^2$ between the top and bottom in the derivative, our function is not differentiable at $x = 0$.)

*Questions*

(1) Differentiate each of these with respect to the indicated variable. Note: you will want to think first about which rule(s) you need to apply, and then apply them. *Don't worry too much about simplifying your answers – any correct form is OK for this.*

(a) $f(x) = \dfrac{x^2 + e^x}{\sqrt{x}}$

(b) $g(t) = e^t\left(1 + \dfrac{t^2}{1+t^2}\right)$

(c) $h(z) = \dfrac{3}{z^{2/3}} - z(e^z + 4z)$

(2) Section 3.4 in our book builds on the way we motivated the study of derivatives by considering instantaneous velocities and slopes of tangent lines. If $f$ is any function and $f'(a)$ exists, then we can think of $f'(a)$ as an *(instantaneous) rate of change* of $f$ with respect to the variable in $f$, at $a$. The *units* of an instantaneous rate of change are always (units of $f$-values)/(units of

the input variable in $f$). For instance, if we had a function $P(R)$ giving the electrical power (in units of watts) delivered to a device by a battery, as a function of the resistance of the device (in units of ohms), then the units of $P'(R)$ would be watts/ohm. So suppose we have a battery delivering power to a device with

$$P(R) = \frac{2.25R}{(R+.5)^2} = \frac{2.25R}{R^2+R+.25}$$

where $R \geq 0$.

(a) What is the instantaneous rate of change of the power with respect to resistance when $R = 3$ ohms? (Give your answer with correct units.)

(b) What is the power delivered to a device with $R = 5$ ohms? What is the instantaneous rate of change of the power with respect to resistance when $R = 5$ ohms? Give each answer with the correct units.)

(c) Is the instantaneous rate of change of the power with respect to resistance ever equal to zero? What does that mean? Generate a sketch of the graph of $P(R)$ ($R$ on the horizontal axis, $P$ on the vertical axis) and show any points where $P'(R) = 0$.

*Assignment*

One writeup of solutions to these problems from each group, due at the end of class.