

MATH 135 – Calculus 1  
The Derivative Product Rule  
October 21, 2016

*Background*

In today's video, we saw that the derivative of a product of functions is given by

$$(f \cdot g)'(x) = f(x)g'(x) + g(x)f'(x),$$

provided that  $f$  and  $g$  are each differentiable at  $x$ . (In other words, if  $f, g$  are differentiable separately, then so is the product, and the derivative of  $f \cdot g$  is given by the formula above.) Today, we want to practice with this new derivative rule

*Questions*

(1) Differentiate by the product rule:

(a)  $(x^2 + 3x + 1)(x^3 + x + 2)$ . Then, expand out the product, differentiate using our more basic rules, and compare your answers. They should be equal.

(b)  $(x^2 + x + 1)e^x$ .

(c)  $e^{2x}$  (How can you break this up as an  $f(x)g(x)$ ?)

(d)  $(x^{1/2} + 3x)(x - 4e^x)$ .

(2) Differentiate using the product rule “in two stages:”

$$(x^2 + 3x)(e^x + x)(x + x^{-2}).$$

(3) Now suppose you have a product  $y = f(x)g(x)h(x)$  with *three factors* in general. If you group the terms like this:

$$f(x)g(x)h(x) = f(x) \cdot (g(x)h(x)),$$

explain why

$$y' = f(x)(g(x)h(x))' + (g(x)h(x))f'(x).$$

Now work out  $(g(x)h(x))'$  by the product rule and rewrite things to see

$$y' = f(x)g(x)h'(x) + f(x)g'(x)h(x) + f'(x)g(x)h(x).$$

(4) Continuing from (3), what is the pattern here? What if you have a product with any number of factors  $y = f_1(x)f_2(x) \cdots f_n(x)$ . What will the derivative of that product look like?