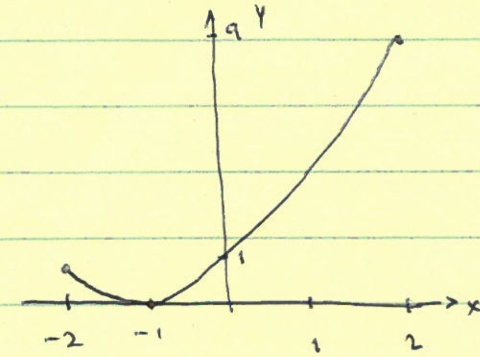
4+4+4+4 = 16 total

MATH 135 Problem Set 1B Solutions

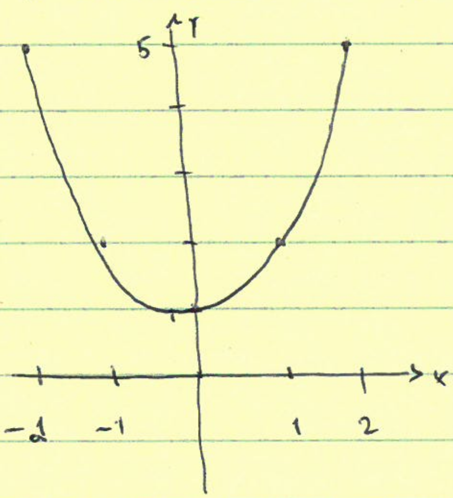
1.1 / 72

(a) $y = f(x+1) = (x+1)^2$, (parabola shifted left 1 unit, but for $x \in [-2,2]$)

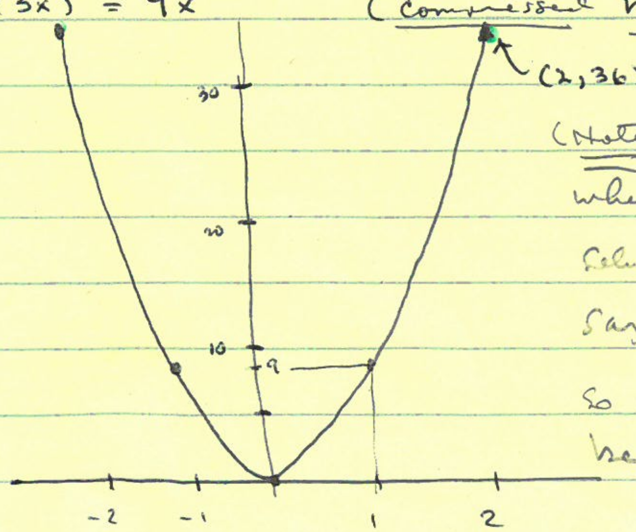


1 point each part
Take off 1 point (out of 4) if the graphs don't show just $x \in [-2,2]$ and explain.

(b) $y = f(x)+1 = x^2+1$, (shift up 1 unit)

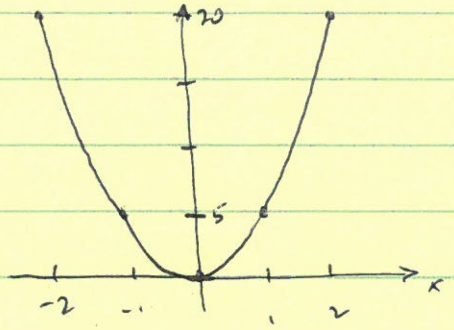


(c) $y = f(3x) = (3x)^2 = 9x^2$ (compressed horizontally)



(Note: I miscopied this when I was writing the solution; the problem actually says $f(5x) = (5x)^2 = 25x^2$ so when $x = \pm 2$, y should be $25 \cdot (2)^2 = 100$.)

(d) $y = 5f(x) = 5x^2$ (stretched vertically)



1.2/21 The line joining $(1,2)$ and $(5,4)$ has slope $\frac{4-2}{5-1} = \frac{2}{4} = \frac{1}{2}$ ① ; the perpendicular slope is -2. ①
The midpoint is $\left(\frac{1+5}{2}, \frac{2+4}{2}\right) = (3,3)$. ① So the perpendicular bisector is $y-3 = (-2)(x-3)$, or

$\boxed{y = -2x + 9}$. ① 4 total

1.2/54 If P is a general point $P = (x,y)$, then $d_1 = \sqrt{(x-0)^2 + (y-\frac{1}{4})^2}$ ① and $d_2 = y + \frac{1}{4}$ ①. So $d_1 = d_2$ says

$$\sqrt{(x-0)^2 + (y-\tfrac{1}{4})^2} = (y + \tfrac{1}{4})$$

$$x^2 + y^2 - \tfrac{1}{2}y + \tfrac{1}{16} = y^2 + \tfrac{1}{2}y + \tfrac{1}{16}$$

$$\therefore \quad x^2 = y \text{ ①}$$

In other words $d_1 = d_2 \Rightarrow P$ is on the parabola.
Conversely, if $y = x^2$, then

$$d_1 = \sqrt{x^2 + (x^2 - \tfrac{1}{4})^2} = \sqrt{x^4 + \tfrac{1}{2}x^2 + \tfrac{1}{16}} \text{ ①}$$
$$= \sqrt{(x^2 + \tfrac{1}{4})^2}$$
$$= x^2 + \tfrac{1}{4}$$
$$= d_2.$$

Also OK if they say the above steps can be reversed to see $d_1 = d_2$ if P is on parabola

So the parabola $y = x^2$ consists of all the points in $\mathbb{R}^2$ for which $d_1 = d_2$.

1.3/40 : In order for the domain to be $\mathbb{R}$, the quadratic $x^2 + 2cx + 4$ should have no real roots.

Method 1: Completing square,

$$x^2 + 2cx + 4 = (x+c)^2 + 4 - c^2 \quad (2)$$

so no real roots $\Leftrightarrow$ $4 - c^2 > 0$ or $-2 < c < 2$, or c in $(-2, 2)$ (2)

either form is OK

Method 2: with the quadratic formula

$$x^2 + 2cx + 4 = 0$$

$$x = \frac{-2c \pm \sqrt{4c^2 - 16}}{2} \quad (2)$$

the roots are not real when $4c^2 - 16 < 0$, or (2) $c^2 < 4$, so $-2 < c < 2$ again.

4 points total, either method is OK