6, 4, 3, 10 $\boxed{\text{Total} = 23}$

MATH 135 Problem Set 9 'B' Solutions

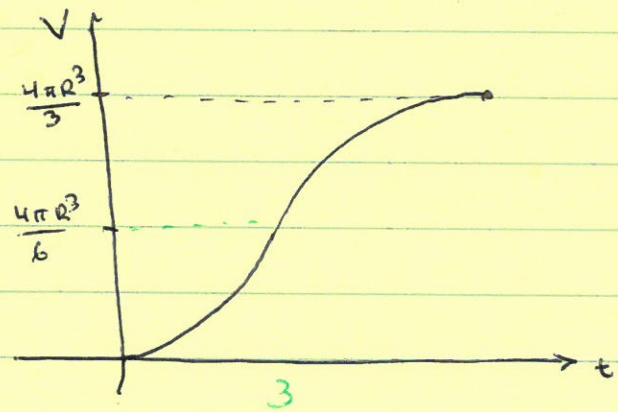
§ 4.4 /.

22. c is the point of inflection; f concave down on (0, c) total 2

23. b, e are the points of inflection; f concave down on (b, e) 2 similar

24. a, d, f are the points of inflection; f concave down on (0, a) and (d, f) 2

63.



$\frac{dh}{dt}$ is constant, so

1 V is growing the fastest when the tank is half-full

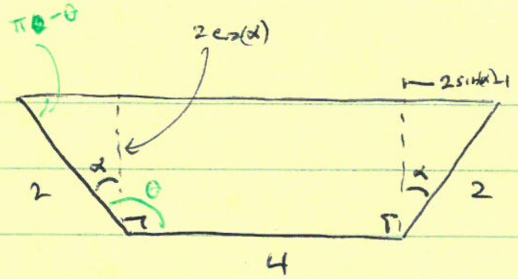
64 (a) $V = \pi\left(Rh^2 - \frac{1}{3}h^3\right) = \pi\left(Rt^2 - \frac{1}{3}t^3\right)$

so $\frac{dV}{dt} = 2\pi R t - \pi t^2$ 1

$\frac{d^2V}{dt^2} = 2\pi R - 2\pi t$ 1, which $= 0$ when $\boxed{t = R}$ 1

this agrees with 63.

4.7/32



(It's also ok if they use θ from diagram in book. then $A(\theta) = 4 \cdot 2\sin(\pi-\theta) + 4\sin(\pi-\theta)\cos(\pi-\theta)$ $= 8\sin\theta \mp 4\sin\theta\cos\theta$.)

Let $\alpha = \theta - \frac{\pi}{2}$. then the dotted vertical segments have length $2\cos\alpha$ and the part outside the rectangle has length $2\sin\alpha$ on both sides. then total area is

$$A(\alpha) = \underbrace{8\cos(\alpha)}_{\text{rectangle}} + \underbrace{4\sin(\alpha)\cos(\alpha)}_{\text{triangles}} \quad 2$$

To maximize the area,

$$0 = A'(\alpha) = -8\sin(\alpha) + 4\cos^2(\alpha) - 4\sin^2(\alpha) \quad 2$$

$$= -8\sin(\alpha) + 4 - 8\sin^2(\alpha)$$

so $\quad 2\sin^2(\alpha) + 2\sin(\alpha) - 1 = 0$

and $\quad \sin(\alpha) = \dfrac{-2 \pm \sqrt{4+8}}{4} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$

Taking − sign gives a value outside the range [−1,1] of sin. so

$$\sin(\alpha) = -\frac{1}{2} + \frac{\sqrt{3}}{2} \doteq .366 \quad 2$$

$$\alpha \doteq \sin^{-1}(.366) \doteq .375 \text{ radians}$$

$$\theta = \alpha + \frac{\pi}{2} \doteq 1.946 \text{ radians} \quad (\text{about } 111.5^\circ) \quad 2$$

this is a maximum because

$$A''(\alpha) = -8\cos(\alpha) - 16\cos(\alpha)\sin(\alpha)$$

$$= -8\cos(\alpha)\left(1 + 2\sin(\alpha)\right)$$

2

< 0 for all $0 < \alpha < \frac{\pi}{2}$.