4, 1, 1, 1, 1, 4, 2 = 14 total

MATH 135 Problem Set 7, part 'B' Solutions

§ 3.7 / 82. $M(t) = \left(a + (b-a)e^{kmt}\right)^{1/m}$

(2)

so $M'(t) = \frac{1}{\cancel{m}}\left(a + (b-a)e^{kmt}\right)^{\frac{1-m}{m}} \cdot (b-a)e^{kmt} \cdot k\cancel{m}$

$$= \frac{k(b-a)e^{kmt}}{\left(a + (b-a)e^{kmt}\right)^{\frac{m-1}{m}}}$$

and $M'(0) = \dfrac{k(b-a)e^{0}}{(a + (b-a)1)^{\frac{m-1}{m}}}$ $\quad (e^0 = 1)$

$$\boxed{M'(0) = \frac{k(b-a)}{b^{\frac{m-1}{m}}}}$$ (2) (or $k(b-a)b^{\frac{1-m}{m}}$)

(1)

87. $\frac{d}{dx} f(g(x))\Big|_{x=6} = f'(g(6)) \cdot g'(6) = f'(6) \cdot 3 = 4 \cdot 3 = \boxed{12}$

(1)

88. $\frac{d}{dx} e^{f(x)}\Big|_{x=4} = e^{f(4)} \cdot f'(4) = e^{0} \cdot 7 = \boxed{7}$

(1)

89. $\frac{d}{dx} g(\sqrt{x})\Big|_{x=16} = g'(\sqrt{16}) \cdot \frac{1}{2\sqrt{16}} = g'(4) \cdot \frac{1}{8} = \frac{1}{2} \cdot \frac{1}{8} = \boxed{\frac{1}{16}}$

90. $\frac{d}{dx} f(2x + g(x))\Big|_{x=1} = f'(2x + g(x))\Big|_{x=1} \cdot (2 + g'(x))\Big|_{x=1}$

$$= f'(2+4) \cdot (2+5)$$

$$= f'(6) \cdot 7$$ (1)

$$= 4 \cdot 7 = \boxed{28}$$

§ 3.8 / 63 $\quad 3x^2 + 4y^2 + 3xy = 24$

$$6x + 8y\frac{dy}{dx} + 3y + 3x\frac{dy}{dx} = 0$$

so $\dfrac{dy}{dx} = \dfrac{-6x - 3y}{8y + 3}$ (2)

this equals 0 when $-6x-3y=0$ so $\boxed{y=-2x}$ 1

the line $y=-2x$ meets the ellipse $3x^2+4y^2+3xy=24$

when $3x^2+4(-2x)^2+3x(-2x) = 24$

$$13x^2 = 24$$

$$x = \pm\sqrt{\frac{24}{13}} \qquad \left(= \pm\frac{2\sqrt{6}}{\sqrt{13}} = \pm\frac{2}{13}\sqrt{78}\right)$$

1

the two points are $\left(\sqrt{\frac{24}{13}}, -2\sqrt{\frac{24}{13}}\right), \left(-\sqrt{\frac{24}{13}}, 2\sqrt{\frac{24}{13}}\right)$ ✓

(from $y=-2x$)

§ 3.9/78 $\frac{d}{dx}\ln(2x) = \frac{1}{2x}\cdot 2 = \frac{1}{x}$ 1 by the chain rule. The easier explanation for this is that $\ln(2x) = \ln(2)+\ln(x)$. Since $\ln(2)$ is constant the derivative is $\frac{d}{dx}\ln(2x) = 0 + \frac{d}{dx}\ln(x) = \frac{1}{x}$. 1