4, 2, 2, 4 = 12 total

MATH 135, Problem Set 6, Part 'B' Solutions

Section 3.3/49 For $f(x) = x^2 e^{-x}$, $f(a) = a^2 e^{-a}$ and $f'(x) = -x^2 e^{-x} + 2x e^{-x} = (2x - x^2) e^{-x}$, so $f'(a) = (2a - a^2) e^{-a}$. the tangent line at $(a, f(a))$ is 1

$$y - a^2 e^{-a} = (2a - a^2) e^{-a} (x - a),$$ 1

this passes through $(x, y) = (0, 0)$ when

$$-a^2 e^{-a} = (2a - a^2) e^{-a} (-a)$$ 1

or $0 = e^{-a} (a^3 - a^2) = e^{-a} a^2 (a - 1)$.

the problem states $a > 0$, so $a - 1 = 0$ and $\boxed{a = 1}$. 1

3.4/34 With t in hours, the angle between the hour hand and the positive x-axis is $-\frac{2\pi}{12}(t-3)$, and the angle between the minute hand and the positive x-axis is $-2\pi(t-3) + \frac{\pi}{2}$ (both in radians)

2 $$\theta(t) = -2\pi(t-3) + \frac{\pi}{2} + \frac{\pi}{6}(t-3)$$

so $\theta'(3) = -2\pi + \frac{\pi}{6} = \boxed{\frac{-11\pi}{6} \text{ radians/hour}}$

Olivia: If people do something radically different on this one, I'll check it; just leave it ungraded.

3.5/40 (A) is $y = f'(x)$

2 (B) is $y = f(x)$

3.6/54 (Note: there's a typo in the problem).
(book had a + here)

$$\frac{d}{dx}\tan(x) = \lim_{h\to 0} \frac{\tan(x+h)-\tan(x)}{h} \quad 1$$

$$= \lim_{h\to 0} \frac{\frac{\tan(x)+\tan(h)}{1-\tan(x)\tan(h)} - \tan(x)}{h} \quad 1$$

$$= \lim_{h\to 0} \frac{\cancel{\tan(x)} + \tan(h) - \cancel{\tan(x)} + \tan^2(x)\tan(h)}{h(1-\tan(x)\tan(h))}$$

$$= \lim_{h\to 0} \frac{\tan(h)}{h} \cdot \lim_{h\to 0} \frac{(1+\tan^2(x))}{1-\tan(x)\tan(h)} \quad 1$$

$$= \lim_{h\to 0} \frac{\sin(h)}{h} \cdot \lim_{h\to 0} \frac{1}{\cos(h)} \cdot \lim_{h\to 0} \frac{1+\tan^2(x)}{1-\tan(x)\tan(h)}$$

$$= 1 \cdot 1 \cdot \frac{(1+\tan^2 x)}{1} \quad 1$$

$$= 1+\tan^2(x).$$

This equals the usual form $\sec^2(x)$ because of the trig. identity $1+\tan^2(x) = \sec^2(x)$.