3, 2, 4, 4 = 13 total

MATH 135 — Problem Set 5, part 'B' Solutions

3.1/48 From the "zoomed" graph (B) it looks as though (16, 4) and (16.09, 4.012) are on the graph. From these we get a secant line with slope

$$m = \frac{4.012 - 4}{16.09 - 16} = .133 \quad (1)$$

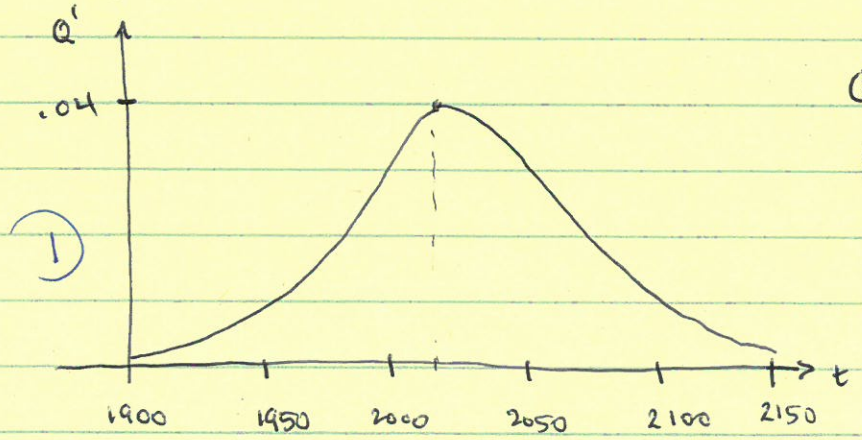
Don't be too picky here anything "reasonable" should get full credit

the actual value of $f'(16)$ comes by computing $f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$ (1), so $f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{8} = .125$ (1)

3.2/44 $g(x) = f'(x)$ because $g(x)$ is negative on $(-1,0)$ where $y = f(x)$ has negative tangent slopes, $g(x) = 0$ at $x = 0$ where the tangent to $y = f(x)$ is horizontal, and $g(x)$ is positive on $(0,1)$ where the tangents to $y = f(x)$ have positive slopes (1) any correct explanation is ok (using signs of f' $\Leftrightarrow$ +/− slopes of tangents)

3.2/46

(a) Roughly, the graph of $Q'(t)$ looks like this



(1)

$Q'(2010)$ estimated from graph ≈ .04 billion barrels per year

(not necessary to estimate max value of Q', just look for general shape.)

(b) Around 2010 (1) don't be too picky (a bit to left of midpoint of [2000, 2025] ⟸ anything in this interval is OK

(c) $\lim_{t\to\infty} Q(t) \doteq 2.3$ billion barrels. This is the "peak" oil production level, predicted by the theory. (1)

(d) $\lim_{t\to\infty} Q'(t) = 0$ (the slope is going to zero as $t \to \infty$) (1)

3.2/83 $f(x) = 9 - x^2$ has $f'(x) = -2x$ (1) by the "shortcut" rules. The tangent line at $(a, 9-a^2)$ has equation (1) $y - (9-a^2) = -2a(x-a)$, or $y = -2a\cdot x + (9+a^2)$. This line contains the point $(5,0)$ when

$$0 = -2a\cdot 5 + 9 + a^2$$

or

$$0 = a^2 - 10a + 9$$
$$= (a-9)(a-1) \quad (1)$$

The point shown has $x \in [0,3]$ so it must be $\boxed{(1,8)}$ (1). (There is also a second point $(9,-72)$ where the tangent goes through $(5,0)$.)