

## MATH 135, Problem Set 4, 'B'

§2.5/52

$$\begin{aligned}
 \lim_{h \rightarrow 0} \frac{\sqrt{a+2h} - \sqrt{a}}{h} &= \lim_{h \rightarrow 0} \frac{(\sqrt{a+2h} - \sqrt{a})}{h} \cdot \frac{(\sqrt{a+2h} + \sqrt{a})}{(\sqrt{a+2h} + \sqrt{a})} \\
 &= \lim_{h \rightarrow 0} \frac{(a+2h) - a}{h(\sqrt{a+2h} + \sqrt{a})} \\
 &= \lim_{h \rightarrow 0} \frac{2h}{h(\sqrt{a+2h} + \sqrt{a})} \\
 &= \lim_{h \rightarrow 0} \frac{2}{(\sqrt{a+2h} + \sqrt{a})} = \frac{2}{2\sqrt{a}} = \boxed{\frac{1}{\sqrt{a}}}
 \end{aligned}$$

§2.6/50

$$\begin{aligned}
 \lim_{\theta \rightarrow 0} \frac{\sin 3\theta - 3 \sin \theta}{\theta^3} &= \lim_{\theta \rightarrow 0} \frac{3 \cancel{\sin \theta} - 4 \sin^3 \theta - 3 \cancel{\sin \theta}}{\theta^3} \quad (\text{given identity}) \\
 &= \lim_{\theta \rightarrow 0} -4 \cdot \frac{\sin \theta}{\theta} \cdot \frac{\sin \theta}{\theta} \cdot \frac{\sin \theta}{\theta} \\
 &= \boxed{-4} \quad \text{by the limit Product Law and } \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1.
 \end{aligned}$$

§2.7/33

$$(a) \quad \lim_{s \rightarrow \infty} \frac{As}{k+s} = \lim_{s \rightarrow \infty} \frac{As}{k+s} \cdot \frac{\frac{1}{s}}{\frac{1}{s}} = \lim_{s \rightarrow \infty} \frac{A}{\frac{k}{s} + 1} = A$$

$$(b) \quad \frac{As}{k+s} = \frac{1}{2} A \quad \text{when} \quad s = \frac{1}{2}(s+k), \quad \text{so} \quad \frac{1}{2}s = \frac{1}{2}k \quad \text{or} \quad \boxed{s=k}.$$

$$(c) \quad \frac{(0.1)s}{1.25+s} = (0.75)(0.1) \quad \text{when} \quad (0.1)s = (1.25+s)(0.075) \quad \text{so} \quad .025s = .09375 \quad \frac{1}{s} = \frac{3.75}{1} \quad \boxed{2}$$