4, 4, 5, 3   Total 16

MATH 135 Problem Set 3, 'B'

2.1/24

(a) the slope of the line A is the average rate of change of the fraction infected from week 4 to week 6. the

2 slope of the line B is the instantaneous rate of change of the fraction infected at time $t=6$ weeks

(b) The slopes of the tangents are increasing on this

1 range, so at $t=3$

(c) At $t=4$ (tangents have decreasing slopes on this range).

1

2.2/64 Consider $L(n) = \lim_{x\to 1}\left[\frac{n}{1-x^n} - \frac{1}{1-x}\right]$.

the problem said to investigate these limits numerically and try to guess a formula for $L(n)$ as a function of the positive integer n. For example, with $n=1$, clearly $L(1) = \lim_{x\to 1}\left[\frac{1}{1-x} - \frac{1}{1-x}\right] = \lim_{x\to 1} 0 = 0$

For $n=2$, we find   give 1 point for each $L(n)$ they estimate correctly, up to a total of 3

.9	.99	.999	.9999	1.0001	1.001	1.01	1.1
.52632	.50251	.50025	.50002	.49998	.49975	.49751	.47619

so it seems likely that $\lim_{x\to 1}\left[\frac{2}{1-x^2} - \frac{1}{1-x}\right] = \frac{1}{2} = .5$

Similarly $L(3) = 1$, $L(4) = 1.5$, etc. So we guess $L(n)$ is a linear function of n:

$$1 \quad L(n) = \frac{n}{2} - \frac{1}{2} \quad \left(\text{or } \frac{n-1}{2}\right).$$

(any equivalent form ok too)

2.4/84 1 each

(a) continuous

(b) Continuous

(c) discontinuous: jumps each time a deposit or withdrawal is made, or an interest payment is credited

(d) discontinuous: jumps when the teacher gets a raise (probably)

(e) discontinuous: Note the actual population is an integer; it increases by 1 with each birth, decreases by 1 with each death. (Note that we often use continuous functions to model a population, though.)

2.2/40

$\lim_{x \to 1^-} f(x) = 3$ $\lim_{x \to 2^-} f(x) = 2$ $\lim_{x \to 4^-} f(x) = 2$

$\lim_{x \to 1^+} f(x) = 1$ $\lim_{x \to 2^+} f(x) = 1$ $\lim_{x \to 4^-} f(x) = 2$

$\lim_{x \to 1} f(x)$ dne $\lim_{x \to 2} f(x)$ dne $\lim_{x \to 4} f(x) = 2$

1 1 1