

MATH 135 – Calculus 1  
Extreme Values  
November 14, 2016

*Background*

Recall from today's video:

- A continuous function on a closed interval attains a *maximum value* and a *minimum value* on that interval.
- Those maximum and minimum values are attained either at endpoints of the interval or at interior points – nothing deep going on there, just logic(!)
- If the maximum or minimum value is attained at a point  $x = c$  *other than an endpoint*, then  $c$  must be a *critical point* of  $f$  – either a solution of  $f'(c) = 0$ , or else a place where  $f'(c)$  does not exist.
- So to find the maximum and minimum value of a continuous function of a continuous  $f(x)$  on a closed interval  $[a, b]$ , we can:
  - (i) Compute  $f'(x)$  and find all critical points  $c$  in  $[a, b]$ .
  - (ii) Compute  $f(a)$ ,  $f(b)$ , and  $f(c)$  for all critical points found in the first step.
  - (iii) Then the maximum value will be the largest of the numbers found in the previous step and the minimum value will be the smallest of those numbers.

*Questions*

For each of the following functions,

- (i) determine all critical points in the given interval,
- (ii) compute the values of  $f$  at the critical points in the interval, and compute the values at the endpoints of the interval,
- (iii) determine the maximum and minimum values of  $f$  on the interval.

This is the process sketched above(!)

1.  $f(x) = x^3 - 12x^2 + 21x$  on  $[0, 11]$ .
2.  $f(x) = (x^2 + 2x)e^{-x}$  on  $[1, 5]$
3.  $f(x) = 5 \tan^{-1}(x) - x$  on  $[-5, 5]$ .
4.  $f(x) = (t - t^2)^{2/3}$  on  $[0, 2]$ . (Note: Careful on this one – you should find critical points where  $f'(x)$  does not exist.)