MATH 135 – Calculus 1
Exam 3 Practice Problems
November 18, 2016

I. Find the indicated derivative(s) and simplify.

(A) y' for
$$y = \ln(x)\left(x^7 - \frac{4}{\sqrt{x}}\right)$$

(B) y' for
$$y = \sin^{-1}(e^{2x} + 2)$$

(C) y' for
$$y = \frac{\ln(x+1)}{3x^4 - 1}$$

(D) y' for
$$y = \frac{\sin(x)}{1 + \cos(x)}$$

(E) y' for
$$y = \tan^{-1}(x^2 + x)$$

(F) Using implicit differentiation:
$$xy^2 - 3y^3 + 2x^4 - 4xy = 2$$

(G) Find the equation of the line tangent to the curve from (F) at $(x, y) = (1, 0)$

(H) Find
$$\frac{d}{dx}\left(5x\sqrt{x} - \frac{2}{x^3} + 11x - 4\right)$$

(I)
$$\frac{d}{dt}\left(\frac{t^2 e^{3t}}{t^4 + 1}\right)$$

(J)
$$\frac{d^2}{dz^2}\left(\frac{z^2 - 2z + 4}{z^2 + 1}\right)$$

(L)
$$\frac{d}{dx}\left(\sin(x)\left(x^7 - \frac{4}{\sqrt{x}}\right)\right)$$

(M) Find y' (note this is just another way of asking the same question!)
$$y = (e^{2x} + 2)^3$$

(N) Find y' and y''

$$y = \frac{x+1}{3x^4 - 1}$$

(O) Find y'

$$y = \frac{\sin(x)}{1+\cos(x)} + x^2\cos(x^3+3)$$

II. The total cost (in \$) of repaying a car loan at interest rate of $r\%$ per year is $C = f(r)$.

A) What is the meaning of the statement $f(7) = 20000$?

B) What is the meaning of the statement $f'(7) = 3000$? What are the units of $f'(7)$?

III. The quantity of a reagent present in a chemical reaction is given by $Q(t) = t^3 - 3t^2 + t + 30$ grams at time t seconds for all $t \geq 0$. (Note: For a question like this, I could also give you the plot of the function and ask questions like those below. In this case you need to start from the formula and compute $Q'(t)$; if you were given the graph, you need to make the connection between slopes of tangent lines and signs of $Q'(t)$ visually.)

(A) Over which intervals with $t \geq 0$ is the amount increasing? (i.e. $Q'(t) > 0$) decreasing (i.e. $Q'(t) < 0$)?

(B) Over which intervals is the rate of change of Q increasing? decreasing? decreasing?

IV. A spherical balloon is being inflated at 20 cubic inches per minute. When the radius is 6 inches, at what rate is the radius of the balloon increasing? At what rate is the surface area increasing? (The volume of a sphere of radius r is $V = \frac{4\pi r^3}{3}$ and the surface area is $A = 4\pi r^2$.)

V. A baseball diamond is a square with side of length 90 feet. After hitting the ball, a player leaves home plate and runs toward first base at 15 ft/sec. (Assume the runner is running straight along the base path – this is a bit unrealistic, of course, but let's keep it simple for the purposes of the problem!) How fast is the runner's distance from second base changing when he is half way to first base?

VI. All parts of this question refer to $f(x) = 4x^3 - x^4$.

(A) Find and classify all the critical points of f using the First Derivative Test.

(B) Over which intervals is the graph $y = f(x)$ concave up? concave down?

(C) Sketch the graph $y = f(x)$.

(D) Find the absolute maximum and minimum of $f(x)$ on the interval $[1, 4]$.

VII. All three parts of this question refer to the function $f(x)$ *whose derivative is plotted in Figure 1. NOTE: This is the graph* $y = f'(x)$ *not* $y = f(x)$.

(A) Give approximate values for all the critical points of $f(x)$ in the interval shown, and say whether f has a local maximum, a local minimum, or neither at each.

(B) Find approximate values for all the inflection points of $f(x)$.

(C) Over which intervals is $y = f(x)$ concave up? concave down?

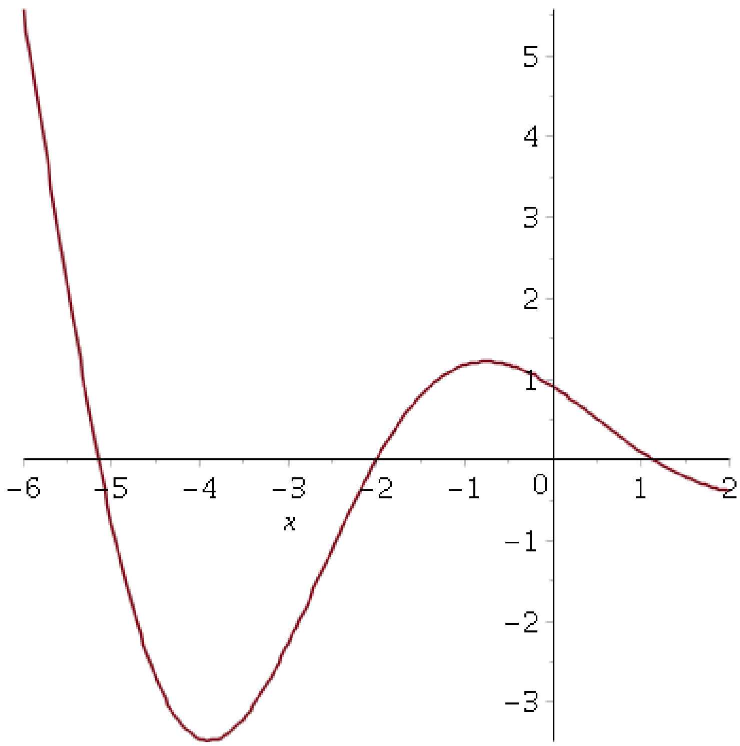

Figure 1: Plot of $y = f'(x)$ for Problem VII