MATH 135 – Calculus 1
Curve Sketching
November 28, 2016

Background

When mathematicians and scientists want to understand graphs of functions and other curves these days, they almost always turn to graphing calculators and mathematical software to generate the plots. So learning to plot functions by hand can seem like a waste of time. However, in my experience, some experience with curve sketching by hand is a great way to solidify your understanding of the First and Second Derivative Tests, concavity, asymptotes, etc. So while you might never need to do this again in "real life," it's still a worthwhile educational exercise.

Here's a checklist of things to do and look for, given $y = f(x)$ to plot:

1. Determine the domain of your function and note whether the graph has any vertical asymptotes. Draw those in if there are any.

2. Compute $f'(x)$ and find the critical points $x = c$, and the critical values $f(c)$.

3. On each interval between adjacent critical points, determine the sign of $f'(x)$. Recall that intervals where $f'(x) > 0$ are intervals where f is increasing and intervals where $f'(x) < 0$ are intervals where f is decreasing.

4. Compute $f''(x)$ and determine any x where $f''(x) = 0$ or where $f''(x)$ does not exist.

5. Determine the intervals where f is concave up and concave down using the sign of f'' (recall that inflection points are places where the concavity changes. If your function has inflection points, find them and the function values there.

6. Determine horizontal asymptotes (if any), by taking $\lim_{x\to\pm\infty} f(x)$. (L'Hopital's Rule will be useful for some of these. One or other of these directions might not make sense for some functions if the domain does not extend to $\pm\infty$.)

7. The critical points and the inflection points can be thought of as *transition points* as described in Section 4.6 of the text.

8. You should now be prepared to sketch the graph using the transition points, the information about where the function is increasing, decreasing, concave up, and concave down. See Figures 1 and 2 on page 231 for the four basic local shapes of graphs and an example of how they can be put together.

Questions

Find the transition points (critical points and points of inflection), intervals of increase and decrease, intervals of upward and downward concavity, asymptotes, and sketch the graph:

1. $y = x^5 - 15x^3$ (Hint: This one has three critical points, three points of inflection, and no asymptotes.)

2. $y = x - 2\ln(x)$ (Hint: $\lim_{x\to 0+} \ln(x) = -\infty$.)

3. $y = \dfrac{1}{x^2 - 2x}$ (Hint: This one has both horizontal and vertical asymptotes. Be sure to simplify $f'(x)$ before computing the second derivative.)

4. $y = x^2 e^{-2x}$ (Hint: This has a horizontal asymptote in one direction but not the other.)