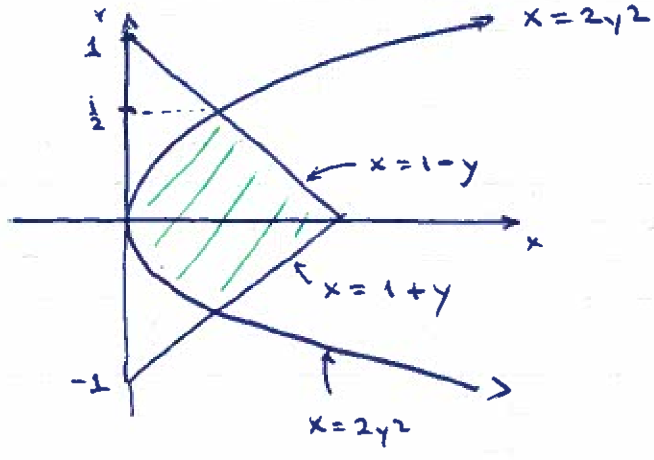
MATH 136, Problem Set 6 'B' Solutions

§ 6.1 / 32



$x \geq 2y^2$

$1 - x - |y| \geq 0$

gives the shaded region

$2y^2 = 1 - y$ when

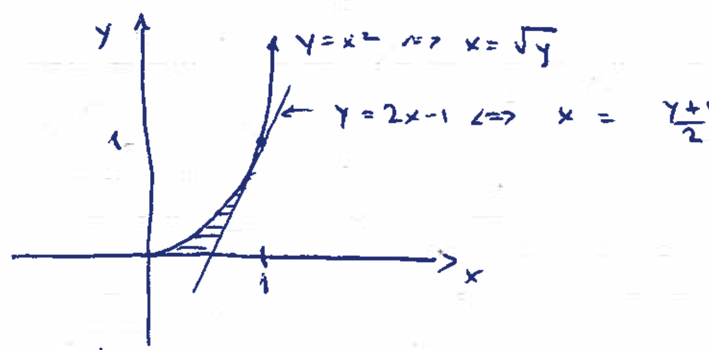
$0 = 2y^2 + y - 1 = (2y-1)(y+1)$

By symmetry, $A = 2\int_0^{1/2} 1 - y - 2y^2 \, dy$

$= 2\left[y - \frac{y^2}{2} - \frac{2y^3}{3} \right]_0^{1/2}$

$= 2\left[\frac{1}{2} - \frac{1}{8} - \frac{1}{12} \right] = \boxed{\frac{7}{12}}$

§ 6.1/40 the tangent to $y = x^2$ at $(1,1)$ has slope $m = \frac{dy}{dx} = 2x\Big|_{x=1} = 2$, so the equation is $y - 1 = 2(x-1)$ or $y = 2x - 1$

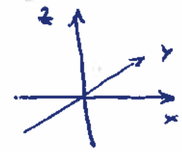


$A = \int_0^1 \frac{y+1}{2} - \sqrt{y} \, dy$ $\left(\text{or } \int_0^{1/2} x^2 \, dx + \int_{1/2}^1 x^2 - (2x-1) \, dx \right)$

$= \frac{y^2}{4} + \frac{y}{2} - \frac{2}{3} y^{3/2} \Big|_0^1 = \frac{1}{4} + \frac{1}{2} - \frac{2}{3} = \boxed{\frac{1}{12}}$

§6.2/48 (see diagrams in the problem)

Say the two cylinders are $\begin{cases} x^2 + z^2 = r^2 \\ y^2 + z^2 = r^2 \end{cases}$



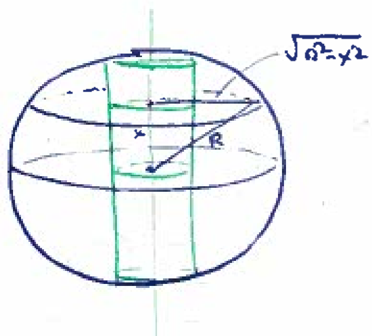
In a <u>horizontal</u> slice $z = h$, $x^2 = y^2$, so $x = \pm y = \pm\sqrt{r^2 - h^2}$. So the cross-sections are <u>squares</u>, with side $2\sqrt{r^2-h^2}$ in the slice at height h. this gives

$$V = \int_{-r}^{r} \left(2\sqrt{r^2-h^2}\right)^2 dh$$

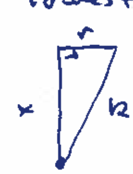
$$= 4\left(r^2 h - \frac{h^3}{3}\right)\Big|_{-r}^{r}$$

$$= 4 \cdot \left(\frac{2r^3}{3} + \frac{2r^3}{3}\right) = \boxed{\frac{16r^3}{3}}$$

52.



Slice <u>horizontally</u> again. In the slice x above or below the center of the sphere, the shape is a "washer" with inner radius r, outer radius $\sqrt{R^2-x^2}$. the highest and lowest nonempty slices are for x like this:



so $x = \pm\sqrt{R^2-r^2}$

$$V = \int_{-\sqrt{R^2-r^2}}^{\sqrt{R^2-r^2}} \pi\left(\sqrt{R^2-x^2}\right)^2 - \pi r^2 \, dx$$

$$= \pi\left[R^2 x - \frac{x^3}{3} - r^2 x\right]_{-\sqrt{R^2-r^2}}^{\sqrt{R^2-r^2}} = \boxed{\frac{4\pi\left(R^2-r^2\right)^{3/2}}{3}}$$