MATH 136 PS 2, 'B' Solutions

5.4/20

(a) $f(x) = g'(x)$ so $\begin{cases} \text{local maxima of } g \text{ at } x = 2, 6 \\ \text{local minima of } g \text{ at } x = 4, 8 \end{cases}$

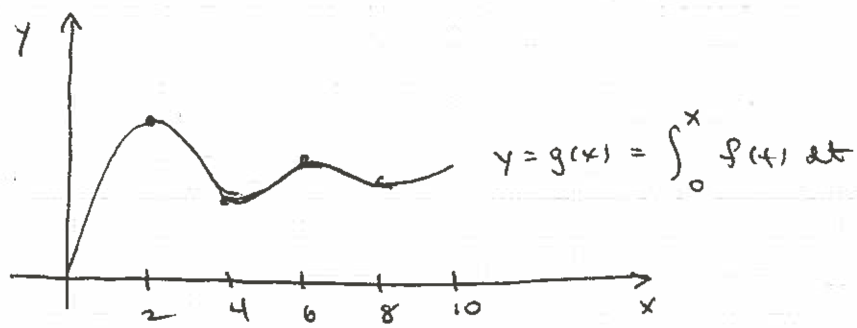
(b) the absolute max is at $x = 2$, since we see from the graph of f:

$$\int_0^2 f(t)\,dt > \left| \int_2^4 f(t)\,dt \right| > \int_4^6 f(t)\,dt > \left| \int_6^8 f(t)\,dt \right| > \int_8^{10} f(t)\,dt$$

(c) g is concave down on intervals where f is decreasing ($g'' = f'$ negative): approx.

$(1,3) \cup (5,7) \cup (9,10)$

(d) the inequalities in b above say $g(2) > g(6)$ and $g(8) > g(4)$:



5.4/31. Take $\frac{d}{dx}$ of both sides and use FTC, part 1:

$$\frac{d}{dx}\left[6 + \int_a^x \frac{f(t)}{t^2}\,dt\right] = \frac{d}{dx}\left(2\sqrt{x}\right)$$

so $0 + \frac{f(x)}{x^2} = \frac{1}{x^{1/2}}$ and $\boxed{f(x) = x^{3/2}}$

then $\int_a^x x^{-1/2}\,dx = 2\sqrt{x} - 6$

$\quad\quad ||$

$2\sqrt{x} - 2\sqrt{a} = 2\sqrt{x} - 6$, so $2\sqrt{a} = 6$ or $\boxed{a=9}$

5.5/61. $A_1 = \int_0^1 e^{\sqrt{x}}\,dx$. Let $u = \sqrt{x}$ so $du = \frac{1}{2\sqrt{x}}\,dx$ and

$dx = 2\sqrt{x}\,du = 2u\,du$. The integral is

$A_1 = \int_0^1 2u e^u\,du$. $A_2 = \int_0^1 2x e^x\,dx$. Then

$A_3 = \int_0^{\pi/2} e^{\sin x} \sin 2x\,dx = \int_0^{\pi/2} e^{\sin x} \cdot 2\sin x \cos x\,dx$ (trig. id.)

Let $u = \sin x$, $du = \cos x\,dx$. This gives

$A_3 = \int_0^1 e^u \cdot 2u\,du$. This shows all three

areas are equal.

5.5/66. Beginning of week 3 is $t=2$; end of

week 4 is $t=4$. So we want

$$\int_2^4 5000\left(1 - \frac{100}{(t+10)^2}\right) dt$$

$= 5000 \cdot \int_2^4 1 - \frac{100}{(t+10)^2}\,dt$ let $u = t+2$ $du = dt$

$\int \frac{1}{u^2}\,du = -\frac{1}{u} + C$

$= 5000\left[t\Big|_2^4 + \frac{100}{(t+10)}\Big]_2^4 \right]$

$\doteq 4047$ calculators.