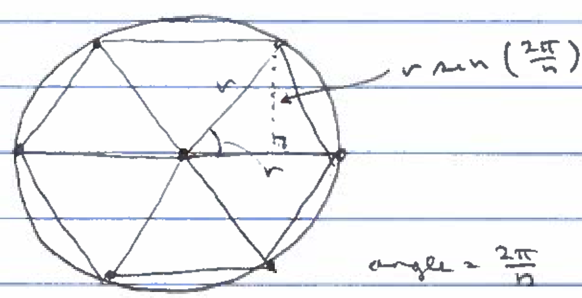
MATH 136 – PS 1 'B' Solutions

§5.1/20. If we take $f(x) = (5+x)^{10}$, then $x_i = \frac{2i}{n}$ for $i = 0, \dots, n$ and $a = x_0 = 0$, $b = x_n = 2$.
So one possible answer is $f(x) = (5+x)^{10}$, $a = 0, b = 0$.
Equally correct would be to incorporate the 5 into the x_i, so $x_i = 5 + \frac{2i}{n}$. Then $x_0 = a = 5$ and $b = x_n = 7$. This limit also gives the area under $y = x^{10}$, above $y = 0$, between $x = 5$ and $x = 7$.

§5.1/28. (a) Here's the picture with $\underline{n = 6}$: a regular hexagon inscribed in the circle of radius r:



The area of 1 triangle is $\frac{1}{2}bh = \frac{1}{2}r \cdot r\sin\left(\frac{2\pi}{n}\right)$
$= \frac{r^2}{2}\sin\left(\frac{2\pi}{n}\right)$. There are n triangles, so

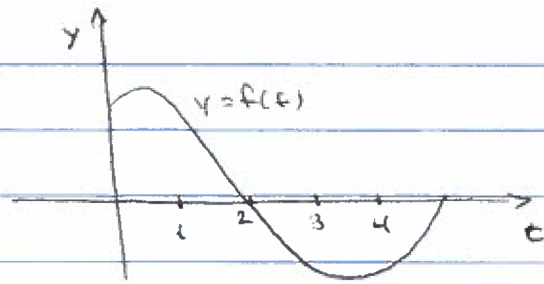
$$A_n = \frac{nr^2}{2}\sin\left(\frac{2\pi}{n}\right)$$

(b) As $n \to \infty$, the triangles will "fill up" the circle. But

$$\lim_{n\to\infty} A_n = \lim_{n\to\infty} \frac{nr^2}{2}\sin\left(\frac{2\pi}{n}\right) = \frac{1}{2}r^2 \cdot \lim_{n\to\infty} \frac{\sin\left(\frac{2\pi}{n}\right)}{\left(\frac{2\pi}{n}\right)} 2\pi$$

As $n \to \infty$, $\frac{2\pi}{n} \to 0$. By work from last semester (or L'Hopital's Rule, $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$. Hence $\lim_{n\to\infty} A_n = \frac{1}{2}r^2 \cdot 1 \cdot 2\pi = \boxed{\pi r^2}$

§5.2/48



We have $F(0) = \int_2^0 f(t)\,dt = -\int_0^2 f(t)\,dt < 0$
($f(t) \geq 0$ here)

$F(1) = \int_2^1 f(t)\,dt = -\int_1^2 f(t)\,dt < 0$
($f(t) \geq 0$ here)

$F(2) = \int_2^2 f(t)\,dt = 0$

$F(3) = \int_2^3 f(t)\,dt < 0$ since $f(t) \leq 0$ here

$F(4) = \int_2^4 f(t)\,dt < 0$ " " ".

Hence $\boxed{F(2)=0}$ is the largest value: all the others are <u>negative</u>.

§5.3/74 We have $A = \int_0^a e^x\,dx$, $B = \int_0^b e^x\,dx$

and $B = 3A$. By the Evaluation theorem,

$e^x \Big|_0^b = 3 \cdot e^x \Big|_0^a$ or $e^b - 1 = 3(e^a - 1)$.

So: $e^b - 1 = 3e^a - 3$
$e^b = 3e^a - 2$

$$\boxed{b = \ln(3e^a - 2)}.$$