MATH 136, section 1 – Calculus 2
Integration By Substitution Practice – Solutions
February 7, 2014

Each of the following integrals can be done by the method of u-substitution:

1. Determine the appropriate function u,
2. Compute du,
3. Change the integral to an equivalent form in the new variable u. If it is a definite integral, you can convert the limits of integration as well.
4. Integrate, then
5. Resubstitute u to express the answer in terms of the original variable (indefinite integral cases), or evaluate (definite integral cases).

- $\displaystyle\int x\sqrt{x^2+16}\,dx$. Let $u = x^2+16$. Then $du = 2x\,dx$ and $x\,dx = \frac{1}{2}du$. So the integral is
$$\frac{1}{2}\int \sqrt{u}\,dx = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = \frac{1}{3}(x^2+16)^{3/2} + C.$$

- $\displaystyle\int \cos\theta e^{1+\sin\theta}\,d\theta$. Let $u = 1+\sin\theta$. Then $du = \cos\theta$ and the integral becomes:
$$\int e^u\,du = e^u + C = e^{1+\sin\theta} + C.$$

- $\displaystyle\int_{\pi/4}^{\pi/2} \cos^3(4\theta)\sin(4\theta)\,d\theta$. Let $u = \cos(4\theta)$. Then $du = -4\sin(4\theta)\,d\theta$ (chain rule) and $\sin(4\theta)\,d\theta = \frac{-1}{4}du$. Also, the limits of integration become $x = \pi/4 \Rightarrow u = \cos(\pi) = -1$ and $x = \pi/2 \Rightarrow u = \cos(2\pi) = 1$. So the integral becomes
$$\frac{-1}{4}\int_{-1}^{1} u^3\,du = \left.\frac{-u^4}{16}\right|_{-1}^{1} = 0.$$

- $\displaystyle\int \frac{1}{\sqrt{1-4x^2}}\,dx$. For this one, note the similarity with the formula for the derivative of the inverse sine function if we think of $\sqrt{1-4x^2}$ as $\sqrt{1-(2x)^2}$. If $u = 2x$, then $du = 2\;dx$ so the integral becomes:

$$\frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1}(u) + C = \frac{1}{2}\sin^{-1}(2x) + C.$$

- $\displaystyle\int_0^{1/4} \frac{x}{\sqrt{1-4x^2}}\,dx$. This one is different from the previous one because of the extra x in the numerator. That x is part of du for $u = 1-4x^2$. With that substitution, the integral becomes

$$\frac{-1}{8}\int_{u=1}^{u=3/4} u^{-1/2}\;du = \frac{-1}{4}u^{1/2}\Big|_1^{3/4} = \frac{1}{4}\left(1-\sqrt{\frac{3}{4}}\right).$$

- $\displaystyle\int \frac{1}{x\ln(x)}\,dx$. Let $u = \ln(x)$, then $du = \frac{1}{x}\;dx$ and the integral becomes

$$\int \frac{1}{u}\;du = \ln|u| + C = \ln|\ln(x)| + C.$$

- $\displaystyle\int \frac{\cos(\sqrt{y})}{\sqrt{y}}\,dy$. Let $u = \sqrt{y}$. Then $du = \frac{1}{2\sqrt{y}}\;dy$, so the integral is

$$2\int \cos(u)\;du = 2\sin(u) + C = 2\sin(\sqrt{y}) + C.$$