MATH 136 – Calculus 2
Group Discussion Day 5 – Applications of differential Equations

I. A rare mineral deposit was formed in the collision of a meteorite with the earth some time long in the past. The deposit originally contained some amount of the Uranium isotope U^{238}, which is radioactive with half-life 4.51×10^9 years, but none of the lead isotope Pb^{207}, which is the end decay product of U^{238}. The ratio of the number of U^{238} atoms to the number of Pb^{207} in the mineral deposit today is 0.9. When did the collision with the meteorite occur?

II. Just before midday, the body of an apparent homicide victim is found in a room that is kept at a constant temperature of 70 degrees F. At 12 noon the temperature of the body is measured to be 80 degrees F and at 1 pm, the temperature has decreased to 75 degrees F. Assume that the temperature of the body at the time of death was 98.6 degrees F and that the body cooled according to Newton's law. When did the murder occur?

III. Suppose that a water tank has a hole at the bottom and water is draining out. If $y(t)$ denotes the depth of the water at time t, then *Torricelli's Law* implies that $y(t)$ will satisfy the differential equation

$$A(y)\frac{dy}{dt} = -a\sqrt{2gy}$$

where

- g is the acceleration of gravity near the surface of the earth ($\doteq 32$ in feet per seconds squared),
- A is the cross-section area of the tank at depth y, and
- a is a constant (it represents the area of the hole).

A) If the tank has the shape of the solid obtained by rotating some curve $x = g(y)$ about the y-axis, determine $g(y)$ so that *the depth of the water in the tank will fall at a constant rate.*

B) For what would such a tank be useful? (Hint: Look up the term *clepsydra* on Wikipedia. An "outflow" clepsydra formed from a tank of this shape would have a special property.)