MATH 136, Section 1 — Discussion Day 5 Solutions

I. Let $\begin{cases} u(t) \text{ be the amount of } U^{238} \\ L(t) \text{ be the amount of } Pb^{207} \end{cases}$ present at time t.

$u(t)$ is undergoing exponential decay with half-life 4.51×10^9 years

So $u(t) = u(0)\, e^{kt}$ where $k = \dfrac{\ln(1/2)}{4.51 \times 10^9} \doteq -1.5369 \times 10^{-10}$.

Take $t=0$ when the meteorite struck the earth. Then at present $\dfrac{u(t)}{L(t)} = .9$. Since each uranium atom decay produces one lead atom, $L(t) = u(0) - u(t)$, so

$$\frac{u(t)}{u(0)-u(t)} = .9, \text{ so } 1.9\, u(t) = .9\, u(0), \text{ or}$$

$$u(t) = \frac{.9}{1.9}\, u(0)$$

$$\frac{.9}{1.9} \cancel{u(0)} = \cancel{u(0)}\, e^{(-1.5369\times 10^{-10})\, t} \quad \text{when}$$

$$t = \frac{\ln(.9/1.9)}{-1.5369 \times 10^{-10}} \doteq 4.86 \times 10^9 \quad (4.86 \text{ billion years})$$

II. Say $t=0$ is noon, and $t=1$ is 1 pm. We have

$$T(t) = A + (T(0) - A)\, e^{kt}$$

$$= 70 + (80-70)\, e^{kt}$$

and setting $t=1$,

$$75 = 70 + 10e^{k}, \text{ so } e^{k} = \frac{5}{10} = \frac{1}{2} \text{ and } k = \ln(\tfrac{1}{2}) \doteq -.6931$$

Now we want to determine when the temperature was 98.6, so we solve $98.6 = 70 + 10\, e^{(-.6931)\, t}$

$$\frac{\ln\left(\frac{28.6}{10}\right)}{-.6931} = t$$

$$\boxed{-1.5 \doteq t}$$

So the murder occurred at about 1.5 hours <u>before noon</u>, or about 10:30 am.

III. $A(y) = \pi(g(y))^2$, so $\frac{dy}{dt}$ is constant $= b < 0$ when

$$\pi(g(y))^2 \cdot b = -a\sqrt{2g} \cdot y^{1/2}$$

$$(g(y))^2 = \frac{-a\sqrt{2g}}{\pi b} \, y^{1/2}$$

So $\boxed{g(y) = (\text{const.})\, y^{1/4}}$, or from $x = (\text{const})\, y^{1/4}$, we also have $\boxed{y = \text{const. } x^4}$.