Mathematics 136 – Calculus 2
Exam 3 – Review Sheet
April 22, 2014

General Information

As announced in the course syllabus, the third midterm exam of the semester will be given in class on Friday, May 2. The format will be similar to that of the first two midterms and the exams from last semester.

- You will be provided with (an extract from) the table of integrals for use on this exam.
- You will not be given a calculator for the exam. If you want to use one, you will need to supply it yourself. Any calculator like the TI-30, which does NOT have graphing capabilities, is OK. (Note: Some of you may have one of these calculators purchased for use in Chemistry courses here. That is also OK.)
- Use of phones, I-pods, and any electronic devices other than your calculator *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack (make sure cell phones are turned off).

What will be covered

The exam will cover the material since the last exam (Problem Sets 7, 8, and 9), namely the following material from sections 6.4 - 6.6, 6.8, 7.1 - 7.5, and 8.2 - 8.5 of Stewart:

1. Additional applications of integration (Chapter 6)
   (a) Arc length of graphs and parametric curves
   (b) Average value of a function
   (c) Center of mass (centroid)
   (d) Probability applications
2. Differential equations (Chapter 7)
   (a) Solutions of a differential equation (know how to determine whether a given function is or is not a solution of a given equation; for this type of question it will not be necessary to derive a general solution),
   (b) direction fields (be prepared to sketch a simple direction field by hand, and/or identify computer-generated plots),
   (c) Euler's method for approximating solutions,
   (d) Separable equations and solution by separation of variables and integration,
   (e) Exponential growth and decay problems; other growth and decay problems (be prepared to set up a differential equation matching a verbal description, solve it, and use the solution to answer questions),

(f) Logistic equations – know the form

$$y' = ky\left(1 - \frac{y}{M}\right)$$

of the logistic equation, the properties of the solutions, and the analytic formula

$$y = \frac{M}{1 + be^{-kt}}$$

for the solutions.

3. Infinite Series (Chapter 8)
   (a) the concepts of convergence for an infinite series,
   (b) key classes of examples such as geometric series, telescoping series, the harmonic series, p-series, etc.
   (c) general convergence tests: the Integral Test, the Alternating Series Test, the nth Term Test for Divergence, the Ratio Test for absolute convergence
   (d) power series, radius of convergence, interval of convergence

Important Note: Some of the problems on this exam (in particular anything from the sections in Chapter 6, plus solving separable differential equations and applying the Integral Test for series) will require you to set up and compute integrals to find the quantity that is asked for. In addition to knowing how to set up the required integral, *any* of the methods of integration tested on the first and second exam (i.e. basic rules, u-substitution, integration by parts, trigonometric substitution, partial fractions, or consultation of a table of integrals) might be required to evaluate the integral. Again, there is a high degree of "cumulativity" built into this material.

There will be a review for the exam in class on Wednesday, April 30.

Review Problems

- Section 6.4/7,11
- Section 6.5/7,11
- Section 6.6/49,51
- Section 6.8/3,5,7
- Section 7.1/1,3,5,9,11
- Section 7.2/1,3-6,9,23
- Section 7.3/1,3,5,7,9,11,33,35
- Section 7.4/3,9,11,13,17

- Section 7.5/1,3,7
- Section 8.2/11,13,15,19,29
- Section 8.3/3,5,7,21 (use a comparison test)
- Section 8.4/5,7, 21, 23, 25
- Section 8.5/5,7,9

Sample Exam Questions

This list is much longer than the actual exam will be (to give you some idea of the range of different questions that might be asked).

I. (A) Set up and evaluate the integral to compute the arclength of the curve $x = 3t^2$, $y = 2t^3$, $0 \le t \le 2$.

(B) Set up and evaluate the integral to compute the arclength of the curve $y = \frac{1}{6}(x^2+4)^{3/2}$, $0 \le x \le 3$. (Hint: the arclength integral simplifies to a manageable form if you are careful with the algebra.)

II. (A) Find the average value of $f(x) = x\sqrt{1+x^4}$ on the interval $[0, 2]$.

(B) Set up the integrals needed to compute the location of the center of mass of a thin plate of constant density in the shape of the region between the graph $y = f(x)$ from part (A) and the x-axis. (Do not try to evaluate them.)

(C) For which c is $f(x) = cx\sqrt{1+x^4}$ a probability density function on the interval $[0, 2]$ (i.e. take $f(x) = 0$ if x is outside $[0, 2]$).

III. (A) Show that for any constant c, $y = x^2 + \frac{c}{x^2}$ is a solution of the differential equation

$$y' = 4x - \frac{2}{x}y.$$

(B) All parts of this question refer to the differential equation

$$y' = y(4 - y)$$

(1) Sketch the slope field of this equation, showing the slopes at points on the lines $y = 0, 1, 2, 3, 4, 5$

(2) On your slope field, sketch the graph of the solution of the equation with $y(0) = 1$.

(3) Use Euler's method to approximate the solution of this equation with $y(0) = 1$ for $0 \le x \le 1$ using $n = 4$.

(4) This is a separable equation, find the general solution and determine the constant of integration from the initial condition $y(0) = 1$. (Note: this is similar in form to a logistic equation.)

(C) Find the general solutions of the following differential equations

(1) $y' = \dfrac{y}{x(x+1)}$

(2) $y' = \dfrac{\sqrt{1-x^2}}{e^{2y}}$.

(D) Newton's Law of Cooling states that the rate at which the temperature of an object changes is proportional to the difference between the object's temperature and the surrounding temperature. A hot cup of tea with temperature 100°C is placed on a counter in a room maintained at constant temperature 20°C. Ten minutes later the tea has cooled to 76°C. How long will it take to cool off to 45°C? (Express Newton's Law as a differential equation, solve it for the temperature function, then use that to answer the question.)

IV. (A) Does the infinite series $\sum_{n=1}^{\infty} n \ln(1+n)$ converge? Why or why not?

(B) Use the Integral Test to determine whether or not

$$\sum_{k=1}^{\infty} \frac{k}{e^k}$$

converges.

(C) Use the Ratio Test to determine whether or not

$$\sum_{k=0}^{\infty} \frac{3^n}{n!}$$

converges.

(D) Determine (with justification!) whether or not the following series converge:

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}, \qquad \sum_{n=0}^{\infty} (-1)^n \frac{3^n}{\pi^{2n}}, \qquad \sum_{n=1}^{\infty} \frac{1}{n^{1.01}}.$$

(E) For each of the given power series, find the interval of convergence. (In particular, give the radius of convergence, and investigate convergence at the endpoints.)

$$\begin{aligned} f(x) &= \sum_{k=0}^{\infty} x^k = \frac{1}{1-x}, \\ g(x) &= \sum_{n=1}^{\infty} \frac{(2x)^n}{\sqrt{n}}, \\ h(x) &= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-5)^n}{n \cdot 3^n}. \end{aligned}$$