Mathematics 136 – Calculus 2
Exam 1 – Review Sheet
February 14, 2014

General Information

As announced in the course syllabus, the first midterm exam of the semester will be given in class on Friday, February 21. The format will be similar to that of the midterm exams last semester.

- You may use a scientific calculator for the exam which does NOT have graphing capabilities (a TI-30 or equivalent). *No calculators will be provided from this point on since our old departmental supply is defunct and will not be replaced.*
- Use of cell phones, I-pods, and all other electronic devices besides a basic calculator *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack (make sure cell phones are turned off).

What will be covered

The exam will cover the material since the start of the semester – Problem Sets 1, 2, 3, including the following material from sections 5.1 through 5.6 of Stewart:

1) Riemann sums and the definition of the definite integral.
2) The Fundamental Theorem of Calculus: Part 1: If $f(t)$ is continuous on $[a,b]$ and $F(x) = \int_a^x f(t)\,dt$, then $F'(x) = f(x)$. Part 2: (the Evaluation Theorem) If $F(x)$ is an antiderivative of a continuous function $f(x)$ on $[a,b]$, then
$$\int_a^b f(x)\,dx = F(b) - F(a)$$
3) Antiderivatives graphically and numerically
4) Basic antiderivative rules: All rules coming from basic derivative formulas: Know $\int x^n\,dx$, $\int a^x\,dx$, $\int \sin(x)\,dx$, $\int \cos(x)\,dx$, $\int \frac{1}{x^2+1}\,dx$, $\int \frac{1}{\sqrt{1-x^2}}\,dx$, and so forth, plus the sum, and constant multiple rules
5) Integrals by substitution
6) Integrals by parts: Selecting appropriate u and dv, computing du and v, using the parts formula $\int u\,dv = uv - \int v\,du$, then finishing the integral on the right. Recall that this might involve using parts again, or another method such as substitution.

Note: Some problems may ask you to carry out a particular integration method on a problem. Others may leave the choice up to you. *Be prepared for both types of questions!* There will be a review for the exam in class on Wednesday, February 19.

Review Problems

The Review Problems 1 - 46 at the end of Chapter 5 are good for preparation for this exam. It's not necessary to work all of them. But you should try a good selection and practice choosing a method at least for most of the integrals in problems 9 - 46.

Sample Exam Questions

Note: The actual exam will be considerably shorter than the following list of questions. The purpose here is just to give an idea of the range of different topics that will be covered and how questions might be posed.

I.

A) Evaluate the Riemann sums for $f(x) = x^2 - 2x$ for $0 \le x \le 1$ using $n = 4$ and $x_i^* =$ right endpoints, then repeat with left endpoints, then repeat with midpoints.

B) In part A), one of your values is definitely larger than the actual value of $\int_0^1 x^2 - 2x\,dx$ and one is definitely smaller than the integral. Which is which? (Answer without calculating the value of the integral and then check your work.)

C) The following limit represents a definite integral:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos\left(\frac{i\pi}{n}\right)}{\frac{i\pi}{n}} \frac{\pi}{n}$$

What is the integral?

II. Let

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 3 \\ x - 2 & \text{if } 3 \le x \le 5 \\ 13 - 2x & \text{if } 5 \le x \le 8 \end{cases}$$

A) Sketch the graph $y = f(x)$. In the rest of the parts, $F(x) = \int_0^x f(t)\,dt$, where f is the function from part A.

B) Compute $F(1), F(2), F(3), F(4), F(5), F(6), F(7), F(8)$ given the information in the graph of f.

C) Are there any critical points of F? If so, find them and say whether they are local maxima, local minima, or neither. If not, say why not.

D) Sketch the graph $y = F(x)$. What happens to the graph if the definition of $F(x)$ is changed to

$$F(x) = \int_2^x f(t)\,dt?$$

III. Find the *derivatives* of the following functions:

A) $f(x) = \int_0^x \sin(t)/t\,dt$.

B) $g(x) = \int_5^{x^3} \tan^4(t)\,dt$.

C) $h(x) = \int_{-3x}^{5x} e^{t^2} \sin(t)\ dt$.

IV.

A) Compute $\int 5x^4 - 3\sqrt{x} + e^x + \frac{2}{x}\ dx$

B) Apply a u-substitution to compute $\int x(4x^2-3)^{3/5}\ dx$

C) Apply a u-substitution to compute $\int_1^2 e^{\sin(\pi x)} \cos(\pi x)\ dx$

D) Compute

$$\int \frac{t^2+1}{t^3+3t+3}\ dt$$

(Hint: How does the bottom relate to the top?)

E) Apply integration by parts to compute $\int x^2 e^{-2x}\ dx$

V. Compute each of the integrals below using some combination of basic rules, substitution, integration by parts. You must show all work for full credit.

A)

$$\int \frac{e^{\sqrt{\sin(x)}} \cos(x)}{\sqrt{\sin(x)}}\ dx$$

B)

$$\int e^x \sin(2x)\ dx$$

C)

$$\int_0^1 \tan^{-1}(x)\ dx$$

D) Use integration by parts to show this reduction formula: If n is a positive integer, then

$$\int x^n e^{ax}\ dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax}\ dx$$

E) Apply the result from part (D) to compute $\int x^4 e^{-2x}\ dx$.