MATH 135 - Problem Set 6 B Solutions

§ 3.2/54. The line $x - 2y = 2$ is $y = \frac{1}{2}x - 1$, so slope is $m = \frac{1}{2}$.

On the graph $y = \frac{x-1}{x+1}$, we have $\frac{dy}{dx} = \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{2}{(x+1)^2}$.

This equals $\frac{1}{2}$ when $\frac{1}{2} = \frac{2}{(x+1)^2}$, or $(x+1)^2 = 4$, so $x+1 = \pm 2$.

Hence $x = -3, 1$. When $x = -3$, $y = \frac{-3-1}{-3+1} = 2$, so the tangent

line is $y - 2 = \frac{1}{2}(x+3)$, or $\boxed{y = \frac{1}{2}x + \frac{7}{2}}$. When $x = 1$, $y - 0 = \frac{1}{2}(x-1)$,

so $\boxed{y = \frac{1}{2}x - \frac{1}{2}}$.

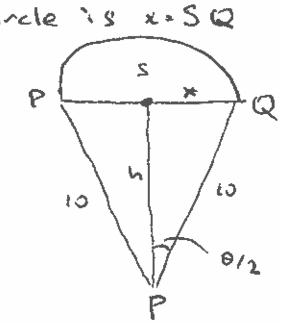
60. (a) By the quotient rule, $\frac{d}{dx}\left(\frac{1}{g(x)}\right) = \frac{g(x)\cdot 0 - 1\cdot g'(x)}{(g(x))^2} = \frac{-g'(x)}{(g(x))^2}$

(b) $\frac{d}{ds}\frac{1}{(s+ke^s)} = \boxed{\frac{-(1+ke^s)}{(s+ke^s)^2}}$

(c) $\frac{d}{dx}(x^{-n}) = \frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{-n\,x^{n-1}}{(x^n)^2} = -n\,x^{n-1-2n} = -n\,x^{-n-1}$.

(So the power rule works for negative integer exponents too.)

§3.3/48 In the figure, let S be the midpoint of PQ.
Then the radius of the semicircle is $x = SQ$ and the height
of the triangle is $h = PS$:



Then $A(\theta) = \frac{1}{2}\pi x^2 = \frac{1}{2}\pi(10\sin(\theta/2))^2 = 50\pi\sin^2(\theta/2)$

$B(\theta) = \frac{1}{2}(2x)h = 10\sin(\theta/2)\cdot 10\cos(\theta/2) = 100\sin(\theta/2)\cos(\theta/2)$.

So $\lim\limits_{\theta\to 0^+}\frac{A(\theta)}{B(\theta)} = \lim\limits_{\theta\to 0^+}\frac{50\pi\sin^2(\theta/2)}{100\sin(\theta/2)\cos(\theta/2)}$

$= \lim\limits_{\theta\to 0^+}\frac{50\pi\sin(\theta/2)}{100\cos(\theta/2)}$ (cancel one $\sin(\theta/2)$)

$= 0$.

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(a) $\lim_{t\to\infty} \dfrac{1}{1+ae^{-kt}} = \dfrac{1}{\lim_{t\to\infty} ae^{-kt} + 1} = \dfrac{1}{0+1} = \boxed{1}$,

since $k>0$. (In the long run, <u>everyone</u> hears the rumor)

(b) By the chain rule $p'(t) = \dfrac{-ae^{-kt}\cdot(-k)}{(1+ae^{-kt})^2} = \boxed{\dfrac{ake^{-kt}}{(1+ae^{-kt})^2}}$.

(C) With $a=10$, $k=.5$ as given, $p(t)=.8$ when

$$.8 = \frac{1}{1+10e^{-.5t}}$$

$$1+10e^{-.5t} = \frac{1}{.8} = 1.25$$

$$\therefore \quad e^{-.5t} = .025$$

so $t = -2\ln(.025)$

$$\doteq 7.37$$