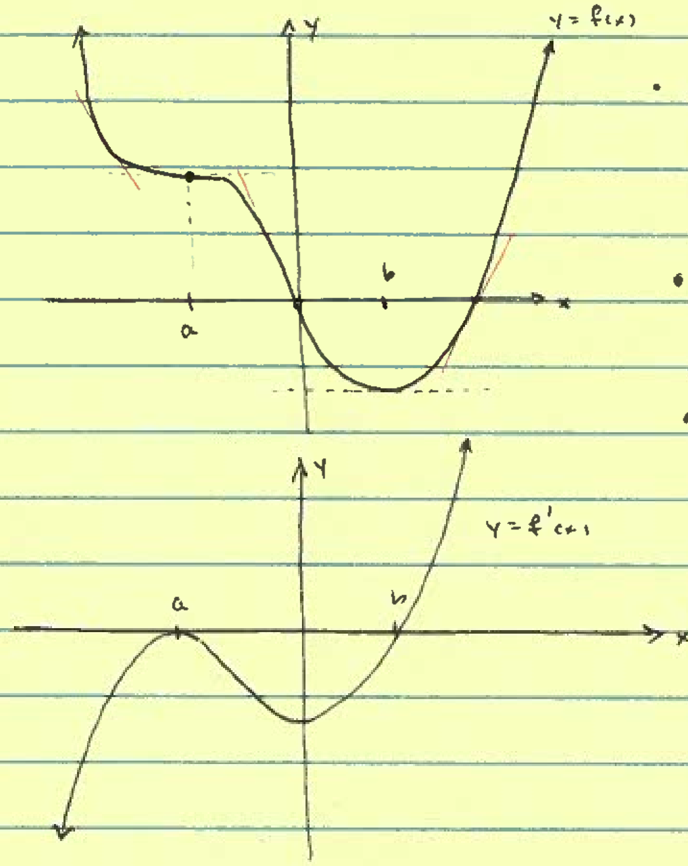
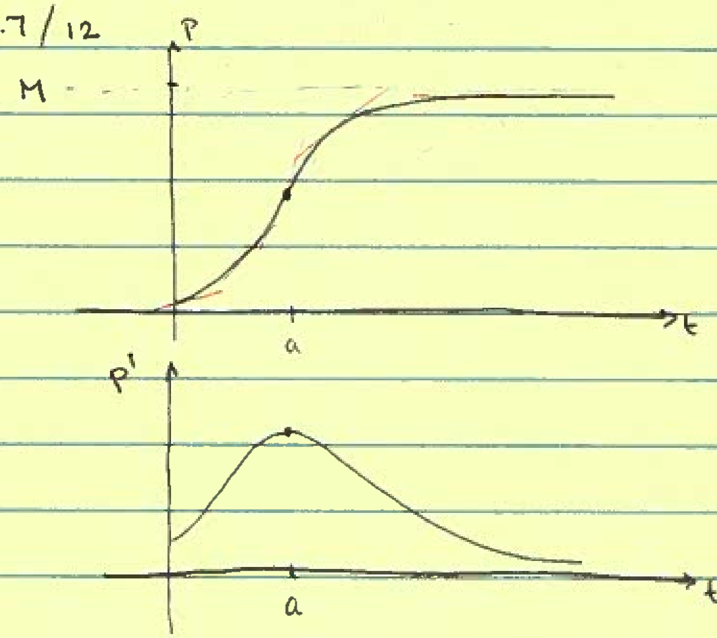
MATH 135 – Problem Set 5 'B' Solutions

2.7/10



- $f'(a) = f'(b) = 0$ since tangent lines are horizontal
- $f'(x) < 0$ on $(-\infty, a) \cup (a, b)$
- $f'(x) > 0$ on $(b, +\infty)$

2.7/12



The yeast population grows faster and faster at the start. But at about $t = 7$, its growth rate hits a maximum and then decreases to near 0.

Comment: In biology, the constant M for which $\lim_{t\to\infty} P(t) = M$ would be called the carrying capacity of the habitat that the yeast culture is grown in. Many real-world populations of living organisms undergo this sort of growth.

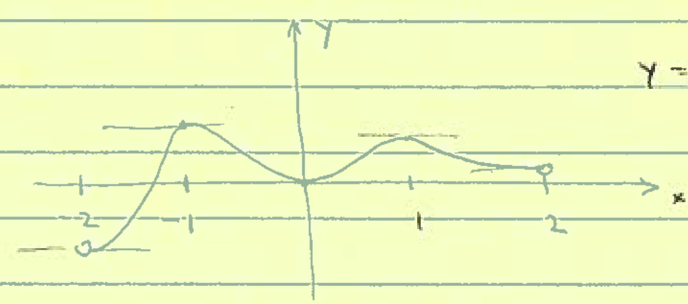
2.8/4

(a) f is increasing on the intervals where $f'(x)>0$:
$(-2,-1) \cup (0,1)$
f is decreasing on the intervals where $f'(x)<0$:
$(-1,0) \cup (1,2)$

(b) f has local maxima at x where $f'(x)$ changes from positive to negative: $-1, 1$
f has local minima at x where $f'(x)$ changes from negative to positive: 0
Note: ± 2 are not included because of the open circles on the graph of f'. We are supposed to assume that the domain of f and f' is the open interval $(-2, 2)$.
So f does not achieve a minimum value as $x \to -2^+$ or $x \to 2^-$. However
$\lim_{x\to -2^+} f'(x) = 0 = \lim_{x\to 2^-} f'(x)$

(C)

$y = f(x)$ with $f(0)=0$.



3.1/66 $\quad y = f(x) = x^4 + ax^3 + bx^2 + cx + d$

$$f'(x) = 4x^3 + 3ax^2 + 2bx + c$$

From the tangent line at $x=0$: $y = 2x+1$, we see $f'(0) = 2$ and $f(0) = 1$ (the tangent must pass through $(0, f(0))$ and have slope equal to $f'(0)$).

$$f(0) = 0^4 + a\cdot 0^3 + b\cdot 0^2 + c\cdot 0 + d = 1$$

$$\Rightarrow \boxed{d=1}$$

$$f'(0) = 4\cdot 0^3 + 3a\cdot 0^2 + 2b\cdot 0 + c = 2$$

$$\Rightarrow \boxed{c=2}$$

$\therefore f(x) = x^4 + ax^3 + bx^2 + 2x + 1$ and $f'(x) = 4x^3 + 3ax^2 + 2bx + 2$

Next, the tangent at $x=1$ is $y = -3x+2$

so $f'(1) = -3$ and $f(1) = -3\cdot 1 + 2 = -1$.

$f'(1) = 4 + 3a + 2b + 2 = -3$, or $\boxed{3a + 2b = -9}$

$f(1) = 1 + a + b + 2 + 1 = -1$, or $\boxed{a + b = -5}$

Solving simultaneously $\boxed{a = 1, \ b = -6}$

So $\boxed{f(x) = x^4 + x^3 + (-6)x^2 + 2x + 1}$