Math 135, Problem Set 4 "B" Solutions

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a) $\frac{x^4-1}{x-1} = \frac{(x-1)(x+1)(x^2+1)}{(x-1)} = (x+1)(x^2+1)$ for $x \neq 1$.

So this <u>does</u> have a removable discontinuity at $x=1$,

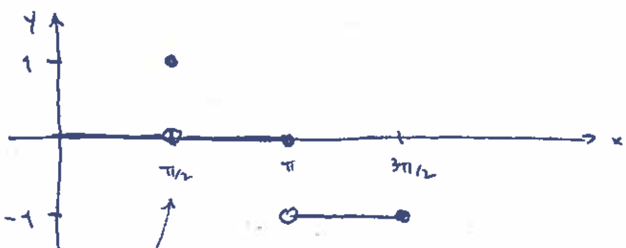
and $g(x) = (x+1)(x^2+1)$

b) $\frac{x^3-x^2-2x}{x-2} = \frac{x(x+1)(x-2)}{x-2} = x(x+1)$ for $x \neq 2$

So this does have a removable discontinuity at $x=2$
and $g(x) = x(x+1)$

c) $[\sin x]$ has a graph that looks like this on

$[0, \frac{3\pi}{2}]$ ([ ] = greatest integer function)



Note: $[\sin x]$ <u>does</u> have a removable disc. at $\frac{\pi}{2}$

Since $\lim_{x\to\pi^-} [\sin x] = 0$, but $\lim_{x\to\pi^+} [\sin x] = -1$, $[\sin x]$ has a jump discontinuity at π, not a removable discontinuity.

40. We are given that f is continuous on $[1,5]$ and the only solutions of $f(x) = 6$ are $x=1$ and $x=4$.

We also know $f(2) = 8$. We claim $f(3) > 6$. <u>Suppose not.</u> Then $f(3) < 6$ since the only solutions of $f(x)=6$ are $x=1$ and $x=4$. But we also have $f(3) < 6 < f(2) = 8$.

So the <u>Intermediate Value theorem</u> (p. 120) would imply $f(x) = 6$ for some $x \in (2,3)$. But this is impossible, because the only solutions of $f(x) = 6$ are $x=1$ and $x=4$. So $f(3) > 6$. //

2.5/ 49. We have $f(x) = \frac{k(x-1)(x+1)}{(x-4)(x+1)} = \frac{k(x-1)}{x-4}$ for $x \neq -1$.

Since $\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{k(x-1)}{x-4} = \frac{2k}{5} = \underset{\text{given}}{2}$, $\boxed{k=5}$

so $f(x) = \dfrac{5(x-1)(x+1)}{(x-4)(x+1)}$ and

(a) $f(0) = \dfrac{5(-1)}{(-4)} = \boxed{\dfrac{5}{4}}$ and (b) $\lim\limits_{x \to \infty} \dfrac{5x-5}{x-4} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}}$

$$= \lim_{x \to \infty} \frac{5 - 5/x}{1 - 4/x}$$

$$= \boxed{5}$$

2.6/

28. Let $f(t) = 2t^3 + t$. Then

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{[2(a+h)^3 + (a+h)] - [2a^3 + a]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2a^3} + 6a^2h + 6ah^2 + 2h^3 + \cancel{a} + h - \cancel{2a^3} - \cancel{a}}{h}$$

$$= \lim_{h \to 0} 6a^2 + 6ah + 2h^2 + 1$$

$$= \boxed{6a^2 + 1}$$

44 (a)

(i) $\left(\begin{array}{c}\text{average rate} \\ \text{of growth} \\ \text{2005 to 2007}\end{array}\right) = \dfrac{15011 - 10241}{2} = 2385$ locations/year

(ii) $(\text{avg. 2005 to 2006}) = \dfrac{12440 - 10241}{1} = 2199$ locations/year

(iii) $(\text{avg 2004 to 2005}) = \dfrac{10241 - 8569}{1} = 1672$ locations/year

(b) average (a) (ii) and (iii):

$$\frac{2199 + 1672}{2} = 1935.5 \text{ locations per year}$$

Note: What we're doing here is equivalent to estimating slope of tangent at a by averaging slopes of secants like this: $\frac{1}{2}\left(\frac{f(a+h) - f(a)}{h} + \frac{f(a) - f(a-h)}{h}\right)$

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$$= \frac{f(a+h) - f(a-h)}{2h}$$