Math 135 – Problem Set 3, part "B"

§ 2.2 /

12. $\lim_{t\to 12^-} f(t) = 150$ and $\lim_{t\to 12^+} f(t) = 300$.

The 150 is the amount of drug in the bloodstream <u>just before</u> the injection at $t=12$. The 300 is the amount <u>just after</u> the injection.

28. Slope is $\lim_{x\to 0} \frac{2^x-1}{x}$. We can estimate this limit from values like this:

x	$\frac{2^x-1}{x}$
.1	.7177
.01	.6955
.001	.6934
−.1	.6697
−.01	.6907
−.001	.6929

$$\lim_{x\to 0} \frac{2^x-1}{x} \doteq .693$$

§ 2.3 /

43. The problem is to <u>show</u> that polynomials are continuous (the "direct substitution rule"). So let $p(x) = c_n x^n + \cdots + c_1 x + c_0$ be a general polynomial. Then

$$\lim_{x\to a} p(x) = \lim_{x\to a} \left(c_n x^n + c_{n-1}x^{n-1} + \cdots + c_1 x + c_0\right)$$

$$= \lim_{x\to a} c_n x^n + \lim_{x\to a} c_{n-1}x^{n-1} + \cdots + \lim_{x\to a} c_1 x + \lim_{x\to a} c_0$$

(limit sum rule)

$$= \lim_{x\to a} c_n \cdot \left(\lim_{x\to a} x\right)^n + \cdots + \lim_{x\to a} c_1 \cdot \lim_{x\to a} x + \lim c_0$$

(limit product rule)

$$= c_n a^n + \cdots + c_1 a + c_0$$

$$= p(a).$$

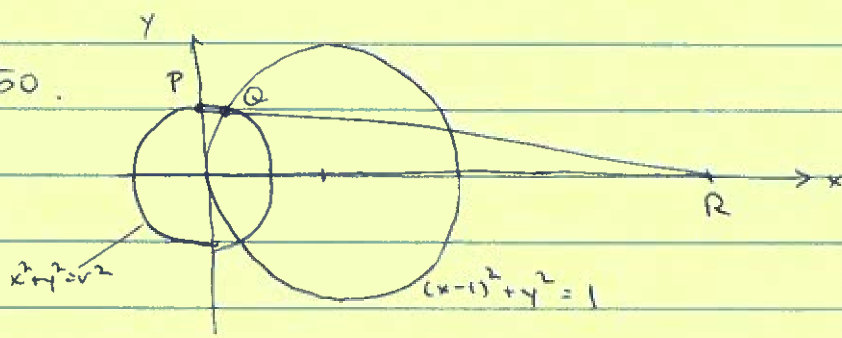
44. A rational function is $r(x) = \frac{p(x)}{q(x)}$ where p, q are polynomials. We have

$$\lim_{x\to a} \frac{p(x)}{q(x)} = \frac{\lim_{x\to a} p(x)}{\lim_{x\to a} q(x)}$$ (limit quotient rule, provided that $\lim_{x\to a} q(x) \neq 0$)

$$= \frac{p(a)}{q(a)}$$ by #43

$$= r(a).$$

50.



By definition, $P = (0, r)$. Say $Q = (a, b)$ then $a^2+b^2=r^2$ and $(a-1)^2+b^2=1$, or $a^2+b^2-2a=0$ so $\boxed{a = \frac{r^2}{2}}$ and $b = +\sqrt{r^2 - \frac{r^4}{4}} = \boxed{\frac{r}{2}\sqrt{4-r^2}}$

Hence the line PQ has equation

$$y = \left(\frac{\frac{r}{2}\sqrt{4-r^2} - r}{\frac{r^2}{2} - 0}\right)x + r = \left(\frac{\sqrt{4-r^2} - 2}{r}\right)x + r$$

this intersects the x-axis when

$$0 = \left(\frac{\sqrt{4-r^2} - 2}{r}\right)x + r,$$

So $x = \dfrac{-r^2}{\sqrt{4-r^2}-2}$ (that is $R = \left(\dfrac{-r^2}{\sqrt{4-r^2}-2}, 0\right)$)

then $\displaystyle\lim_{r\to 0^+} x = \lim_{r\to 0^+} \frac{-r^2}{\sqrt{4-r^2}-2}$

$$= \lim_{r\to 0^+} \frac{-r^2}{\sqrt{4-r^2}-2} \cdot \frac{\sqrt{4-r^2}+2}{\sqrt{4-r^2}+2}$$

$$= \lim_{r\to 0^+} \frac{\cancel{-r^2}\left(\sqrt{4-r^2}+2\right)}{\cancel{-r^2}}$$

$$= \lim_{r\to 0^+} \left(\sqrt{4-r^2}+2\right)$$

$$= \boxed{4}.$$