MATH 135, section 1 – Calculus 1
Solutions – ‘B’ Section, Problem Set 1

1.1/24: The amount of air needed to inflate the balloon from a radius of r to a radius of $r+1$ is the difference of volumes of the two spheres:

$$V(r+1)-V(r)=\frac{4\pi}{3}(r+1)^3-\frac{4\pi}{3}r^3=\frac{4\pi}{3}(3r^2+3r+1)=4\pi(r^2+r)+\frac{4\pi}{3}.$$

(Note: this is ignoring the fact that the air can be compressed as it is introduced into the balloon.)

1.2/18.

a. We are looking for a line in the form $C=md+b$. The line has slope $m=\frac{460-380}{800-480}=\frac{1}{4}$. The line goes through the point $(d,C)=(460,800)$, so $460=\frac{1}{4}(800)+b$, and $b=260$. Answer: $C=\frac{1}{4}d+260$.

b. Substitute $d=1500$ in the formula from part a: $C(1500)=\frac{1}{4}(1500)+260=\$635$.

c. Graph omitted – it's just a straight line through the points $(480,380)$ and $(800,460)$. The everyday meaning of the slope is that this is the marginal cost per mile – the cost of driving the car one additional mile, or $C(d+1)-C(d)$. (This is similar to 1.1/24 above!)

d. The intercept $C(0)=260$ is the monthly cost of owning the car if it is not driven at all. (This would be called the fixed cost, including things like the registration charge, insurance costs, etc.)

e. This is a suitable model because after the fixed costs are accounted for, it is reasonable to assume that the additional costs of driving will be proportional to the number of miles driven. For instance, the cost of gasoline will be roughly proportional to the distance driven and the same will be true for cost of replaceable items like tires, etc. that tend to wear out at a constant rate.

1.5/29

a. After 15 hours the population will be $P(15)=100\cdot 2^{15/3}=3200$

b. After t hours, the population will be $P(t)=100\cdot 2^{t/3}$.

c. After 20 hours, $P(20)=100\cdot 2^{20/3}\doteq 10159$ (I dropped the decimal places since this is supposed to be a whole number(!))

d. Graph omitted: It's an increasing exponential graph going through $(0, 100)$. $P(t) = 100 \cdot 2^{t/3} = 50000$ when $t = 3\frac{\ln(500)}{\ln(2)} \doteq 26.9$ hours.

1.6/45. On the graph, $y = 3 \times 12 = 36$ inches when $x = 2^{36}$ inches. Converting to miles, this is about

$$2^{36} \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \doteq 1,084,588 \text{ mi}.$$

1.6/60.

a. Because t and Q have specific meanings and different units, finding the inverse function in this case just means solving the equation $Q = Q_0(1 - e^{-t/a})$ for t as a function of Q:

$$\begin{aligned} \frac{Q}{Q_0} &= 1 - e^{-t/a} \\ e^{-t/a} &= 1 - \frac{Q}{Q_0} \\ -t/a &= \ln\left(1 - \frac{Q}{Q_0}\right) \\ \therefore t &= -a \ln\left(1 - \frac{Q}{Q_0}\right). \end{aligned}$$

b. When $Q = .9Q_0$ and $a = 2$, this gives $Q = -2\ln(1 - .9) = \ln(100) \doteq 4.6$ seconds.