MATH 135, section 1 – Calculus 1
Lab Day 1 Demo – Parametric Curves

Background and Goals

Suppose a particle is moving along a curve in the xy-plane. Then, at a time t the particle will be at a point on the curve with coordinates (x, y), where x and y are two functions of t, $x = f(t)$ and $y = g(t)$. The two equations for x and y are called *parametric equations* with *parameter* t. Today's lab will show you several additional examples of this idea and give you a way to visualize how curves are traced out this way.

There is *no assignment* connected with this first lab, but you will be creating and saving a worksheet as a record of what you did. The goal is to use the software to demonstrate some of the ideas we were discussing in class on Monday and work through some more complicated examples this way.

There is some general information about Maple available on the course homepage. The main features you will need will also be demonstrated in the lab class meeting. *Lab Demonstration*

A. Launch Maple and open a worksheet. Save it under the name `LabDay1`. At the top, enter your name and today's date in a text region.

B. Consider a moving particle whose coordinates at time t are $x = 2t + 1$ and $y = 3t + 2$, for all real values of t. When $t = -3$, for instance, we have $x = 2(-3) + 1 = -5$ and $y = 3(-3) + 2) = -7$, so the point $[-5, -7]$ is one location of the moving particle.

1) Calculating by hand, find the coordinates $[x, y]$ of the particle at times $t = -2, -1, 0, 1, 2$.

2) Then plot the points $[x, y]$ to guess the curve, using a Maple command of the following format:

```
plot( -your point list- , style=point,symbol=circle);
```

   where `-your point list-` is the list of $[x, y]$ coordinates you generated, in the form `[[x1,y1],[x2,y2],[x3,y3],[x4,y4],[x5,y5]]`. (Note the outer set of square brackets which defines the whole list of five points.) In a text region in your Maple worksheet, answer this question: What curve do you expect these points lie on?

3) Parametric curves can also be plotted in Maple. Type

```
plot([2*t+1,3*t+2,t=-2..2]);
```

   Of course, you may also change the interval for t.

4) To see how the particle traverses the curve, we will use the animation feature of Maple. Type

```
with(plots):

animatecurve([2*t+1,3*t+2,t=-10..10],frames=30);
```

After entering the command you will see a set of axes. Click on the picture. At the top of the screen you will see a button labelled $\triangleright$. Click on it. You will see the curve being traced. You can also look at the curve being traced one piece at a time by clicking repeatedly the button to the right of the $\triangleright$ button.

5) What happens if we replace t by $-t$ in the parametric equations?

6) For after the lab: You probably recognize the curve here, but it is not presented in the usual form of an x, y equation. Solve for t from the equation $x = 2t + 1$ and substitute into the equation $y = 3t + 2$. What is the equation you get and how does it relate to what you saw before?

C. Now we will study the motion of a particle with coordinates given by

$$x = 4\cos(t) \qquad y = 3\sin(t)$$

1) First reason without Maple's help (i.e. use pen and paper). Combine x and y to eliminate the t using a trigonometric identity. What is the shape of the curve traversed by the particle? For which interval of t values does the particle traverse the curve once?

2) Use this interval for t to plot the parameterized curve. (The Maple syntax for π is `Pi`.) Then animate the plot. In what direction is the curve being traversed?

3) What happens if you change t to $-t$ in the parametric equations?

4) What if you change t to $2t$?

5) What if you change t to $t/3$?

6) In each case, how long does it take the particle to go once around the curve?

7) What if you just change the interval for t (say, you have $0 \le t \le 10\pi$)? The graph produced by Maple in this case may look "strange!" Can you explain what happened? Can you fix it so the graph is mathematically correct?

D. Some additional interesting examples: For each $n = 2, 3, 4, 5$, animate the motion of an object moving along the curve

$$x = \cos(t) \qquad y = \sin(nt), \qquad 0 \leq t \leq 2\pi.$$

Write a short paragraph describing the effect of varying n on the curve above and enter it in a text region. (The patterns described by these curves are called *Lissajous figures*.)