MATH 135 – Calculus 1
The Derivative Function – Practice Problems
October 4, 2013

1. Consider the function $f(x) = \frac{x^3}{3} - x$.

a. Compute $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

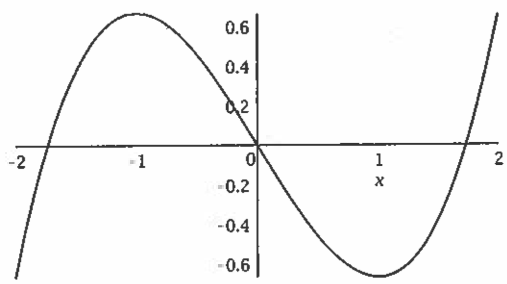
$$f'(x) = \lim_{h\to 0} \frac{\left[\frac{(x+h)^3}{3} - (x+h)\right] - \left[\frac{x^3}{3} - x\right]}{h}$$

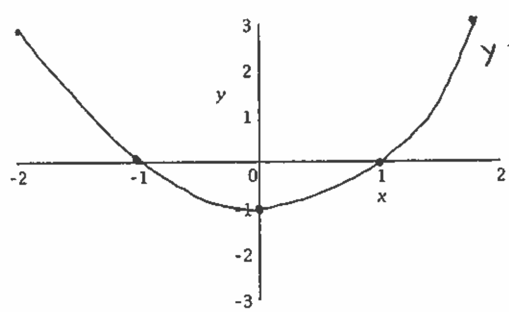
$$= \lim_{h\to 0} \frac{\cancel{\frac{x^3}{3}} + x^2h + xh^2 + \frac{h^3}{3} - \cancel{x} - h - \cancel{\frac{x^3}{3}} + \cancel{x}}{h}$$

$$= \lim_{h\to 0} x^2 + xh + \frac{h^2}{3} - 1$$

$$= \boxed{x^2 - 1}$$

b. Here is a plot of $y = f(x)$. On the empty set of axes below plot $y = f'(x)$.





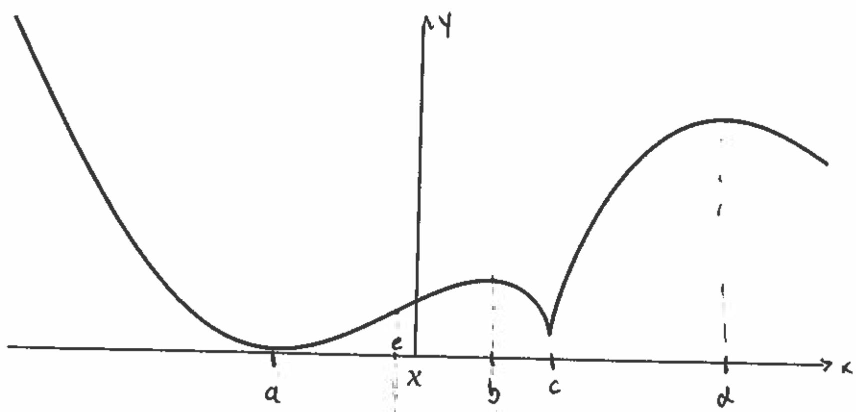
$y = f'(x) = x^2 - 1$

c. Circle the correct answers: On the interval $(-\infty, -1)$, the derivative $f'(x)$ is *positive*/*negative* and $f(x)$ is *increasing*/*decreasing*. On the interval $(-1, 1)$, the derivative $f'(x)$ is *positive*/*negative* and $f(x)$ is *increasing*/*decreasing*. On the interval $(1, +\infty)$, the derivative $f'(x)$ is *positive*/*negative* and $f(x)$ is *increasing*/*decreasing*.

d. What happens on $y = f(x)$ when $f'(x) = 0$? Tangent line is horizontal (slope = 0)

e. (Harder – we have not discussed this!) What happens on the graph $y = f(x)$ when the tangent line to $y = f'(x)$ is horizontal? Here, it's the steepest negative slope; the graph $y = f(x)$ also changes from "concave down" ⋂ to "concave up" ⋃

2. Given the graph $y = f(x)$ below, make a *qualitative* sketch of $y = f'(x)$ on the empty set of axes below. Show any places where $f'(x)$ changes sign and places where $f'(x)$ fails to exist.



Note: here it looks as though slope $\longrightarrow -\infty$ as $x \to c^-$ and $\longrightarrow +\infty$ as $x \to c^+$