MATH 135, section 1 – Calculus 1
Review Sheet – Exam 3
November 13, 2013

General Information

- The third exam of the semester will be given in class on Friday, November 22.
- Our stock of working TI-30X scientific calculators is dwindling, so I recommend you obtain and get familiar with your own calculator for the exam. Any model like the TI-30X or similar ones which do NOT have graphing capabilities is OK. (Note: Some of you may have one of these calculators purchased for use in Chemistry courses here. That is also OK.)
- Use of phones, I-pods, and all other electronic devices except for an approved calculator *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack for the duration of the exam (and make sure cell phones are *turned off*).

What will be covered

The third exam covers the material from Problem Sets 6,7,8: sections 3.2 - 3.8 in Chapter 3 (not 3.9 since we did not cover that at all in class or the problems), plus sections 4.1, 4.2, 4.3, 4.5 in Chapter 4. This includes specifically:

1. Derivatives of trigonometric functions (section 3.3)
2. The chain rule for derivatives of compositions (section 3.4)
3. Implicit differentiation (section 3.5)
4. Derivatives of inverse trigonometric and logarithm functions via implicit differentiation/the chain rule (section 3.6). The method of logarithmic differentiation for functions of the form $u(x)^{v(x)}$ and other forms (section 3.7).
5. Know the idea that derivatives give rates of change from section 3.8.
6. Related rates problems (section 4.1)
7. Critical numbers, maximum and minimum values of $f(x)$on $[a, b]$, the first and second derivative tests, applications to curve sketching (sections 4.2 and 4.3)

How to prepare

You should go over the homework problems as well as the text and your class notes. Many of the problems and questions we discuss in class are excellent examples of test questions. I have also listed some sample problems from the text below. Answers can be found in the text.

Gopal Yalla from the Calculus Workshop will run an optional evening review session on Wednesday, November 20. Come prepared with questions.

Some suggested additional review problems

Chapter 3 Review Problems (pages 248-250)

- 1 - 35 odd numbers. (Note that all the different functions and methods of differentiation that we discussed are included in these, *often combined in various ways.* There will be several similar functions to differentiate on the actual exam and you will need to be able to determine which method to use just by looking at the form of the function.)
- 41, 43, 51, 63, 65
- 68 (*Answers:* (a) $v(t) = 3t^2 - 12$ and $a(t) = 6t$. (b) upward when $t > 2$, downward when $0 \leq t < 2$. (c) Total distance is 23 (7 up and 16 down). (e) Speeding up for $t > 2$, slowing down for $0 < t < 2$.),
- 69 (don't be afraid of the c, ω, δ – they are just arbitrary constants).

Chapter 4 Review Problems

- Graphing, maxima and minima, etc. : 1,3,5,7,9,13
- Related rates: 36 (*Answer:* $\frac{8}{9\pi}$ cm/sec when the depth of the water is 5 cm), 37, 39

Some Sample Exam Questions

Disclaimer: As always, the actual exam questions may be posed in different ways, may be formatted differently, and may focus on somewhat different aspects of the material we have covered. Also, the actual exam will be significantly shorter than these questions(!)

I. Find y' and simplify.

(a)
$$y = \ln(x)\left(x^7 - \frac{4}{\sqrt{x}}\right)$$

(b)
$$y = (e^{2x} + 2)^3$$

(c)
$$y = \frac{x+1}{3x^4 - 1}$$

(d)
$$y = \frac{\sin(x)}{1 + \cos(x)}$$

(e)
$$y = \tan^{-1}(e^{5x})$$

(f)
$$xy^2 - 3y^3 + 2x^4 = 2$$

(g)

$$y = \cos(x)^{x^3}$$

II. The quantity of a reagent present in a chemical reaction is given by $Q(t) = t^3 - 3t^2 + t + 30$ grams at time t seconds for all $t \geq 0$.

(a) Over which intervals with $t \geq 0$ is the amount increasing? decreasing?
(b) Over which intervals is the rate of change of Q increasing? decreasing? decreasing?

III. A spherical balloon is being inflated at 20 cubic inches per minute. When the radius is 6 inches, at what rate is the radius of the balloon increasing? At what rate is the surface area increasing? (The volume of a sphere of radius r is $V = \frac{4\pi r^3}{3}$ and the surface area is $4\pi r^2$.)

IV. All parts of this question refer to $f(x) = 4x^3 - x^4$.

(a) Find and classify all the critical numbers of f using the First Derivative Test.
(b) Over which intervals is the graph $y = f(x)$ concave up? concave down?
(c) Sketch the graph $y = f(x)$.
(d) Find the absolute maximum and minimum of $f(x)$ on the interval $[1, 4]$.

V. All three parts of this question refer to the function $f(x) = x^{2/3} - \frac{1}{5}x^{5/3}$.

(a) Find all the critical numbers of $f(x)$.
(b) Find all the inflection points of $f(x)$.
(c) For which of the critical numbers here is the Second Derivative Test applicable? Why? Determine the type of each such critical number using the Second Derivative Test.